The Politics of Mathematics: Modelling the Economics of Thermal Refits in the German Building Regulations

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Abstract

Experts in the natural sciences are frequently commissioned by governments to provide scientific input for policy development. But it is impossible to do this without importing scientists' own values and political commitments into their models and reports, since their advice has to be tailored to interface with a social setting and cultural context. This paper argues that it is not just science that comes with a set of embedded values, but also mathematics. It examines the mathematical model, developed by physicists, which underlies the German government's claim that the thermal refit measures demanded in its building regulations are 'economic'. It finds the model is permeated with values and political commitments, not merely in its choice of parameters and the values it assigns them, but also in the way its internal algebra works. It explores the ways in which such models hide political commitments, and proposes an alternative style of modelling, which requires citizens to identify and import their own values and commitments into a model designed to resonate more closely with their actual needs.

Key words: mathematical modelling; thermal renovation; German building regulations

Introduction

In liberal democracies a disparate community from various walks of life provide input into policy formation (Callon, et al., 2009 [2001]; Dryzek and Dunleavy, 2009; Fischer, 2003; Hajer and Wagenaar, 2003; Healy et al., 2003; Sabatier and Jenkins-Smith, 1993). This includes not only elected and appointed officials, but also professional associations, NGOs, scientific experts, ad hoc lobby groups, the news media, academia, and individuals who are either interested in or feel themselves affected by the policy in question.

This paper concerns the role of 'experts' within this process, and in particular the mathematical aspects of the work of those whose area of expertise is the physical sciences.

The role of physical science experts can be seen in the context of a wider movement in government since the end of the Second World War, in which specialists covering a range of disciplines were brought in to the process of understanding the effects of policy. One of the founders of modern policy analysis, Harold Lasswell, advocated a mixed methods, interdisciplinary approach, in which sociological study of the human actors, beneficiaries and targets of policy would be combined with more positivist analysis of 'hard' and quantifiable elements of policy domains (Lasswell, 1951). This approach to policy analysis has developed as an extension of the apparatus of government, particularly in the United States but also in Britain and, to varying degrees, in all liberal democracies, in order to improve the formation, implementation and effectiveness of policy (Lynn, 1999).

At first it might seem that the input of physical science experts would and should be politically neutral, providing only technical, matter-of-fact information that can be channelled along the lines chosen by elected representatives in consultation with interest groups affected by the policy. After all there is a long tradition in the physical sciences of separating 'facts' from 'values'. Logical positivism, a movement in the philosophy of science that developed between the world wars, attempted to formalise this for all forms of scientific investigation (Ayer, 1952). It maintained that all real knowledge '…was scientific knowledge – that is, restricted to the observation of facts, to logical inference, and to the determination of regular relationships among facts' (Torgerson, 1986: 36). Values were to be rigorously excluded.

But such an understanding does not accord well with the way politics works. To begin with, there is something mercenary about giving one's value-free expertise to a regime that could do anything it likes with it. People who do this could become, in Douglas G. Hartle's words, 'guns for hire' (Hartle, 1976: 24), blind to 'political reality' (Torgerson, 1986: 37). Increasing a government's knowledge of, say, nuclear physics or germ mutations could be seen as a political act, as it makes that government and its will more powerful. Hence an apparently neutral stance by an expert 'can be grasped as an illusion which tends to suppress critical questions about the political context in which policy analysis is applied' (*ibid:* 38).

But this can be looked at from another angle. If politics is about competing values, then scientific experts can be seen as political animals through and through, as they, being human, have values. Unless the advice they give is restricted to the bald operation of laws of nature, such as Ohm's law or the behaviour of zinc at absolute zero, it will have a social context and therefore involve values. Citing the example of models developed for predicting energy use in the United States in the 1970s, Fischer notes that:

> … it became obvious to even the casual observer that, as deLeon (1988: 70) put it, 'the putatively "objective" nature of the modelling exercises and their computational opaqueness concealed the reality that their underlying and usually unspoken political and social assumptions were what actually "drove" the results'. (Fischer, 2003a: 10)

The ubiquity of values, argues Fischer, is an essential element in a researcher's frame of reference and needs to be properly acknowledged.

Science and Technology Studies explores more closely what scientists actually do when they gather their facts and form their theories (Barnes, 1977; Bloor; 1973, 1999; Latour, 1987; Pickering, 1984). Here the emphasis is on the sociology of the laboratory. These social theorists are interested in what kind of values, social practices and discourses lead scientists to their particular presentations of 'truth'. Hence scientists who are commissioned by governments to produce knowledge on issues affecting policy are often treading a perilous line between natural laws, such as the absorption of infra-red radiation by CO2 molecules, and the social interfaces of such knowledge, such as how much global warming is good for us. This is where values come back into the picture.

We therefore have to look closely at expert reports so as to identify the values that might be driving the logic and colouring the conclusions. In Fischer's words, we have to 'go beyond the empirical data' and examine the practical judgements the expert has used to interface his or her data with the social world (Fischer, 2003: 126).

This kind of analysis is now common in policy studies. This paper takes the process one step further and asks how it might apply to the *mathematics* that gets used in expert reports. On the surface, mathematics seems to be a value-neutral process. While the causes of hurricanes and the behaviour of radioactive waste may be considered from many perspectives, exponentials and algebra seem impervious to value manipulation and interpretive flare. This paper suggests otherwise.

I will examine a mathematical model that lies at the basis of German policy on the minimum legal standards for thermal renovation of existing homes. I will first outline the background to these standards. Then I will show how this model works, and identify the value-judgements inherent in it: its choice of parameters, the numbers it assigns to these, and the way it configures their mathematical relationships with each other. I will then compare it with an alternative model, developed by myself, which is also informed by value judgements (which I will identify), and which produces quite different results. If such results were accepted, this would indicate that the building regulations for thermal renovation are problematic. Finally, I will discuss the implications of these findings for our understanding of what mathematics is, in a social and political context.

This paper covers a small area of a much wider research project on the German government's attempts to get people to insulate their homes. Some of the social understandings in what follows are gleaned from interviews with policy actors, building industry personnel or homeowners, and will be referenced in footnotes.

1. Thermal renovation and the German building regulations

German regulators use two basic parameters for setting and measuring the thermal quality of a new build. The first, H_T, is the average *heat transmission* through the 'thermal envelope' (the outer shell of the building), measured in Watts per square metre of the building envelope per degree Kelvin difference between indoor and outdoor temperatures (W/m^2K). The second, Q_T, is the *quantity of energy* consumed to keep the building at around 20°C all year round, measured in kilowatt hours per square metre of floor area per year (kWh/m^2a). The maximum permissible levels for each of these parameters has been progressively reduced since thermal regulations first came into force in 1976. In the new *Energieeinsparverordnung* (EnEV – Energy saving regulations), introduced in October 2009, Q_T is around 70 kWh/m^2a and H_T is between 65 and 40 W/m^2K, though these vary according to the size and shape of the building (Galvin, 2010).

As from 2002, homes being repaired or renovated must conform to the EnEV regulations. However, if the entire building is being refitted they may consume 40% more energy than an equivalent new build. Hence the current standard for a complete thermal refit of an existing home is around 100 kWh/m^2a, though partial refits must match new build standards.

Such refits are extremely expensive, and often the physical shape of the building has to be changed so as to accommodate the thick insulation and avoid thermal bridges[1]. Hence the economics of thermal renovation is a major issue. Homeowners are legally obliged to meet the EnEV thermal standards when doing even minor repairs. For example they have to insulate an entire wall to new-build standards, or the whole house to refit standards, if just 10% of one wall is being repaired. The government has therefore attempted to keep the standards within what is 'economical' - the German word is *wirtschaftlich* - meaning that the cost should pay for itself, through fuel savings, over the lifetime of the renovations. Hence in preparation for the 2002 regulations and the tightening of these in 2009, the government commissioned studies as to the economics of thermal renovation. The lead author of these studies, Professor Wolfgang Feist, is one of Germany's leading building physicists and the founder and director of the *Passivhaus Institut*. The mathematical model he developed for this study (Feist, 1997; 1998) has now become the standard, with minor variations, for calculating the economics of thermal renovation. It is also widely used in the building industry and is taken as a given in professional discourse regarding thermal renovation.

Calculating the economics of thermal refits is no simple matter, since there is an open set of possible benefits that can be conceived as accruing from a thermal upgrade. These include energy savings, increased comfort and health, improvements to the building structure, better weatherproofing, enhanced (or spoiled) appearance, higher resale or rental value, and the social benefits of reduced CO2 emissions and employment (Martinaitis et al., 2007). However, most established methods of working out refit economics include only the direct gain through energy savings in their models, as this is seen as a direct and quantifiable monetary payback from the investment.

These established methods fall under one of four headings: simple payback time; net present value; internal rate of return; and cost of conserved energy (Martinaitis et al., 2004). The model underlying the German regulations is based on the cost of conserved energy. It compares the cost of the renovations with the cost the homeowner would have paid for the fuel he or she has now saved, over the lifetime of the renovations.

2. How the model works

The model is based on a set of assumptions as to how the physics of energy and buildings interfaces with social realities. The first assumption is that the costs of refurbishment can be divided into two parts: the 'anyway' costs and the 'additional thermal' costs. The 'anyway' costs relate to parts of the job that do not directly contribute to thermal improvement, such as replacing roof tiles after insulating the roof, or re-applying wall render after attaching external wall insulating material. The 'additional thermal' costs relate only to the parts of the job that directly improve the thermal quality – such as purchasing and attaching the insulating material. Only the 'additional thermal' costs are counted in the calculation of whether a job is *wirtschaftlich.*

The second assumption is that a thermal renovation job will have a 'lifetime', after which it will need to be done again. Hence the model is structured as a cost-benefit analysis: if the cost of the job is less than or equal to the financial benefits it is expected to bring over its lifetime (translated into current values), then the job is deemed to be *wirtschaftlich.*

The third assumption is that heating fuel will rise in price by an average annual percentage nominated by the modeller. The fourth is that the homeowner has a certain discount rate, again at a percentage nominated by the modeller.

The fifth assumption concerns the amount of energy expected to be saved annually as a result of the refit. The physics of the building is assumed to determine this entirely.

The model produces figures for costs and benefits in terms of euros per kilowatt-hour (€/kWh), rather than total euros. A project is deemed to be *wirtschaftlich* if the *benefits*, i.e. the value of each kWh of accumulated fuel savings over the lifetime of the renovations, are higher than the cost of saving each kWh (Feist, 1998; Enseling and Hinz, 2006: 22; Kah and Feist, 2005: 9;)[2]. Typical results are, for example, benefits of 0.075 Euros per kWh and costs of 0.062 Euros per kWh – a project that would be deemed *wirtschaftlich.*

3. The parameters in the model

Five parameters are fed into the model, one for each of the assumptions outlined above: the cost of the job; the expected lifetime of the renovations; the trajectory of the future price of heating fuel; the homeowner's discount rate; and the quantity of heating fuel expected to be saved annually.

3.1 The cost of the job

The cost of the job is always calculated as only the 'additional thermal' costs. Since all other costs are seen as providing ancillary benefits – such as a new roof, a new render on the walls, a new balcony, etc. – they should not be counted as expenses contributing to thermal improvement. This is understandable when a run-down house has to be comprehensively renovated 'anyway' to become habitable.

However it is controversial in most other cases. For example, if a house is in good condition but the owners want to upgrade its thermal quality, they will usually have to extend the roof overhang to cover the 16 cm of external wall insulation that is legally required. They might also have to raise the roof to prevent a thermal bridge at the junction of the roof and wall. The costs of roof modifications are not included in the 'additional thermal' costs, as these do not contribute directly to thermal quality. The owners will also have to erect scaffolding and seal the insulating material with a render, but these costs, too, are excluded, as the render does not enhance the thermal quality but is seen as adding value to the house due to its weather resistance.

Replacing windows with the latest thermal models is also not counted fully in the cost-benefit analysis. It is argued that old windows – even if double-glazed - have to be replaced anyway, as they are outmoded. Hence only 10 percent of the cost of the new windows is regarded as an 'additional thermal' cost, and the labour costs of replacement are not included at all.

The result is that the 'additional thermal' costs are much lower than the total costs, for almost every thermal upgrade. The typical cost of a complete thermal upgrade is €500-€1000 per square meter of living area, while typical 'additional thermal' costs account for just €200-€300 of this.

Hence the physicist-mathematician has brought assumptions into the model based on social values. Whether old windows 'have to be replaced' for reasons apart from thermal quality is a question of values, not of physics (unless they are broken or have rotten frames). Remodelling a roof to accommodate thick wall insulation can be seen as a necessary condition of doing thermal improvements, not just as a general improvement for the house. Hence the mathematics used by the physicist produces a far lower figure for this parameter in the model than the real-life costs that a homeowner would actually incur. While a typical refit may cost €100,000, often less than €50,000 will go into the model as 'additional thermal' costs.

3.2 The expected lifetime of the renovations

The second parameter is the expected lifetime of the renovations. One school of thought, represented by the *Institut Wohnen und Umwelt* (IWU – Institute for Housing and Environment), takes this to be 25 years (e.g. Enseling and Hinz, 2006). Another, represented by the Passivhaus Insititut, now assumes a 20-year lifetime with a residual value (e.g. Kah and Feist, 2005). Here the renovations are assumed to remain as good as new for 20 years, and then fall to a lower worth, which lasts and continues to diminish for a further 30 years. So the latter method entails a 50-year planning horizon compared to 25 for the former, though the cost and benefit balances turn out to be comparable. In both cases, the mathematical modelling depends crucially on this time-frame assumption.

A difficulty here is that very few private homeowners think in terms of a timeframe of 25 years – let alone 50 years – to get their money back on improvements in their home. Survey data indicates that only 3% of homeowners have a time-frame longer than 12 years, while 8% would tolerate 8-11 years and 47% expect a 5-7 year payback time (Friedrich, et al. 2007: 35). Large commercial rental housing providers have longer-term horizons, though they seldom plan as far as 20 years ahead[3]. But the model requires homeowners to assess their investment-and-return strategies on such a timeframe. If the physicist-mathematician can prove that they will get their money back within 25 years, then they are legally bound to accept that the upgrade is *wirtschaftlich* and therefore is required by law if they wish to do minor repairs.

3.3 The future cost of heating fuel
The assumption in the prominent models is of an annual increase of 5% to 6% in the cost of heating fuel over the next few decades. This also accords with informal discussion among German policy actors and homeowners, and with sample calculations produced by the German Energy Agency[4]. This is a political choice, since no-one knows the future, but is uncontroversial[5].

3.4 The homeowner's discount rate
The discount rate is an estimate of the annual percentage cost, to the homeowner, of investing a sum of money in a home improvement project. It involves not just measurable elements, such as the annual interest rate paid on a loan, but also 'opportunity costs' and 'risk factors' (Boardman et al., 1986; HM Treasury, 2010: 79-99).

'Opportunity costs' are the costs of lost opportunities for alternative investment due to investing a sum of money in one fixed project. If I invest €100,000 in a thermal refit, I cannot invest that money in, say, shares in a wind power company, which may bring a higher annual return, or my son's education, which my bring both measurable and intangible returns.

It is difficult to put a value on these 'lost opportunities.' Nevertheless they are not included in the mathematical model for determining whether a job is *wirtschaftlich,* whereas they are normally included in cost-benefit analyses, and interviews showed they are well understood by Germany policymakers.

The second part of one's personal discount rate is 'risk factors'. Whenever we invest in some project there is a risk that it will turn out badly. We might die before we reap the financial benefits. The refit might not last the expected 25 years. Due to large internal migration movements within Germany, house prices in my street might fall, thus reducing the value-added from my refit. A homeowner losing her job might not be able to pay the mortgage taken out to do the refit, and could lose her house[6]. The fuel price might not rise as expected, rendering one's investment less profitable. Woodpeckers might bore holes in our polystyrene external wall insulation (Poroton, 2009; Handwerk, 2008; Pecht, 2009), or, as happened recently in Augsburg, martens might invade the roof and literally eat their way through roof insulation[7]. More commonly, a home that previously had no mould and condensation problems might develop these after a thermal refit if the job is not done with exceptional skill.

Neither risk nor lost opportunity costs are incorporated into the mathematical models. Instead, the figure for the discount rate is based on the subsidised mortgage interest rates offered by the German Development Bank[8] plus the annual cost of living increase, and is usually around 3%-5%. By contrast, my interviews with private homeowners indicated they allowed for personal discount rates of around 9%, while the survey data mentioned above indicates typical personal discount rates of around 8%-20%. The international housing firm Grosvenor use a discount rate of around 7%[9]. The higher the discount rate, the more the job costs. Using a low discount rate reduces the apparent cost of the job and makes it more likely to appear *wirtschaftlich.*

3.5 Annual heating fuel savings
To work out the annual heating fuel savings we have to know the fuel consumption before and after the refit. It is much easier to know the consumption after a refit than before, because after the refit one can begin to monitor fuel use and indoor temperature. BASF, based in Ludwigshafen, has a subsidiary which manages its worker housing estates, periodically refurbishes them, and installs sensors to monitor the results[10]. So do some private housing providers, such as *Erbbauverein* in Cologne[11]. The German Energy Agency keeps a database of some 300 refitted homes, many of which are also monitored[12].

Nevertheless, insulating to EnEV 2009 standards will not necessarily give a particular homeowner the theoretical fuel consumption that was calculated from the design methodology required in EnEV 2009. One house may consume, say, 80 kWh/m^{2}a, while another, of apparently identical design, consumes 160 kWh/m^{2}a. Studies show there is a wide range of actual fuel consumption figures, depending on the heating habits of the occupants (Schuler et al., 2000), and the most extreme example in my own research showed a range of 45 to 197 kWh/m^{2}a among identically renovated dwellings in the same apartment block. Clearly the actual energy consumption depends on indoor lifestyle to a large extent. However on average, measured data indicates that refitted homes are at least as good as their refits are designed to be.

It is much harder to get data on what a home was like before a refit. Often this cannot be gleaned from fuel bills because many German apartments were not metered individually until very recently, and much of the new metering is crude and inexact[13].

Further, there is much evidence that homes are kept warmer after refits than before, as people reason that they get more degrees Celsius per euro. Investigations of this so-called 'rebound effect' (Berkhout et al., 2000; Sorrell et al., 2008) indicate that up to 30% of the energy saving through renovation is taken back as extra consumption to achieve more thermal comfort (Greening, et al., 2000).

Modellers therefore use a figure based solely on the physics of the building: how much energy would have been consumed to keep its indoor temperature at around 20°C all year round. This provides objectivity, but makes the model less relevant to a particular household who have to decide whether or not to renovate. For example, a thrifty family who wear warm clothes indoors might make only half the theoretical fuel savings from a refit, as they started from relatively low fuel consumption. It also carries the unfairness that if such a family want to do minor repairs, the law requires them to do a major thermal refit, on the grounds that this will be *wirtschaftlich* for them – even though it their case it will not be.

In summary, the values for all five parameters in the official model have a social dimension, yet at least four of these are set according to criteria that do not allow for the range of values that are expressed in household ownership and maintenance. Quite apart from the internal mathematical workings of the model, the values that prime it are those of the physicists and other science specialists who present the model, not of the consumers of thermal renovation.

4. The internal workings of the model – algebra and politics

As noted above, the principle of the model is to compare the *cost of saving each kilowatt-hour (kWh) of fuel* with the *value of each kWh of the fuel you are expecting to save.* These amounts are averaged over the 25-year lifespan[14] and compared. If the value of each kWh of fuel saved is greater than the cost of saving it, your project is deemed to be *wirtschaftlich.* The modelling proceeds as follows:

Firstly[15], you take your total 'additional thermal' costs, together with the discount rate, and work out what the annualised cost of the refit would be if the cost were spread over 25 years. To do this you use the standard table mortgage formula:

$$\mathbf{P = A \times I/100 \times (1 + I/100)^{25} / ((1 + I/100)^{25} - 1)}$$

where I is your percentage interest (plus cost of living) rate,
A is the 'additional thermal' costs of the project,
and P is the equivalent annual payment of additional thermal costs.

You then work out the cost of each kWh of energy saved, using the formula:

$$\mathbf{C = P / (L \times R)}$$

where L is the living area, in m^2,
R is the reduction in energy use, in kWh per m^2, due to the renovations,
P is the equivalent annual payment of additional thermal costs (see above),
and C is the cost of each kWh of energy saved, in €/kWh.

You now take the current price per kilowatt-hour of heating fuel, 'H'. You then take the expected percentage annual increase in the price of fuel, 'E', and work out each year's fuel price by multiplying H by (1 + E/100) raised to the power of 0, then 1, then 2, etc, up to 24. On a graph this would give an exponential curve.

Next, you work out the average of these 25 values. This gives you 'F', the average price of fuel over the 25-year lifetime of the renovations, again in € per kWh.

If F is greater than C, the project is deemed to be *wirtschaftlich:* the value of each kilowatt-hour of fuel saved is greater than the additional thermal cost of saving one kilowatt-hour of fuel.

Aside from the assumptions built into the model, it also carries a mathematical quirk, which can fool people into thinking their project will pay back earlier than it actually does. When you average an exponential sequence, you get a value that the sequence does not reach until somewhat after its half way point – in this case around 15 years. But your cost 'curve' is flat, as it is the cost of the project averaged over 25 years. Figure 1 shows this on a graph. The higher flat line is the average fuel cost saving over 25 years, while the exponential curve is the actual saving. The flat line does not draw level with the curve until the 15th year. This case is deemed to be *wirtschaftlich* because the cost of saved fuel is lower than the average price of the fuel savings. But the fuel savings do not reach their averaged value until some 2 ½ years after the halfway point. This tends to mask the fact that, while you are paying your average annual costs every year from day one, your fuel cost savings will not reach this level until well in to the future. This is not a problem for a professional housing supplier who thinks long into the future. But a householder may easily be misled by the apparently high value of ‘average’ fuel savings.

In this sense the algebra itself becomes ‘politicised’. The time at which the average value is reached is the solution, for t, to the equation:

$$[a\,(1+f/100)^0 + a\,(1+f/100)^1 + a\,(1+f/100)^2 + \quad + a\,(1+f/100)^{24}] / 25 = a\,(1+f/100)^{t+1}$$

> where a is the initial fuel price and f the annual percentage increase in fuel price.

The equation ‘politicises’ the mathematics in that it gives a solution that is of significance to the homeowner but is hidden behind the parameters the model produces, namely cost and price per kWh over the 25 year lifespan. This is of course not to suggest that algebra itself has political intentions or agency. However it can be seen as an ‘actor’, as understood in actor-network theory (Latour, 2005), in that it influences outcomes quite independently of human intentions. It is not the model designer’s politics that produce this quirk in the model, it is the way the algebra behaves. But still the modellers are responsible for choosing it and not making its effects explicit.

The graph in Figure 1 also shows that for the first 15 years you are losing money, compared to if you had not renovated (because the exponential curve is below the ‘cost of saved fuel’ line). From a purely economic point of view it would have made more sense to wait 15 years and then renovate. You would then be making profits from the day the renovations are complete. But this fact is masked by the way the results of the modelling are presented. The exponential curve is never included in graphical presentations of the model.

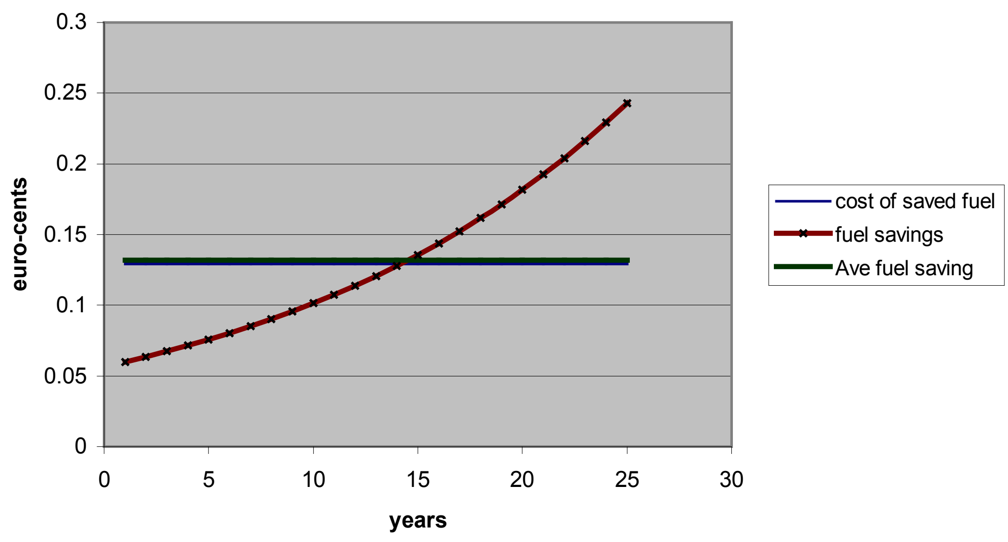

Figure 1 Annual repayments and savings per kWh saved

Hence there are six quite distinct ways in which the values, or political commitments, of the modellers are driving the model:

1. The choice of the type of model used.
2. The choice of parameters used to construct the model.
3. The figures chosen for each parameter, including the rationale for choosing these figures.
4. The criterion chosen to decide whether a refit is *wirtschaftlich:* it must pay back within the 25-year time frame (or 20 years plus residual).
5. The quirk of averaging an exponential sequence, in which the average value is not reached until after the halfway point.
6. The way the model's results are presented, so that it is not made clear that the homeowner is running at a loss in all the years up until the annual benefits reach the level of the annual costs.

5. An alternative model

Here I present a model that requires the homeowner to make more value decisions as to what numbers to use for reach parameter. It uses four parameters rather than five: it makes no prior assumptions about payback time, but instead *calculates* the payback time for a given project. Further, it avoids the exponential-flat curve problem by refraining from constructing a hypothetical average value of fuel price savings (it can do this because it is not working to a set timeframe). This model works as follows:

First, you decide how much of the cost of your project you will count as 'additional thermal' costs. This has to be your own decision because only you know what features of your house you would have repaired 'anyway[16].' This is 'A'.

Second, you work out how much money you expect to save, in fuel costs, per year, by doing the refit. This will depend on your own household habits as much as on the characteristics of the building. In the official methods, this figure is given to you by experts, on the basis of the thermodynamic characteristics of your home. But it depends, in the real world, on your lifestyle and heating habits. This figure is 'S'.

Thirdly, you make an educated guess as to the annual percentage increase in the price of heating fuel. 7% is a safe bet because that is the historic increase since 1973, but it is you who have to live with the consequences of your choice. This figure is 'E' (a percentage).

Fourthly, you work out your personal discount rate. This is usually taken as the likely long term interest rate, plus the annual cost of living increase, plus a portion for risk and a portion for lost opportunities - bearing in mind that this is not the kind of investment you can pull your money out of, if life's circumstances change. This figure is 'D' (a percentage).

You now have to do two sums, using a calculator or spreadsheet. Firstly you work out a factor, 'F', that combines the effect of your estimated fuel price increase and your discount rate. This is:

$$F = (1 + E/100) / (1 + D/100)$$

Then you work out 'N', which is the number of years it will take your project to pay back, using the formula:

$$N = \log [(A/S) \times (F-1) + 1] / \log F$$

A further advantage of this model is that if 'E', your figure for the percentage annual fuel price rise, is the same as 'D', your discount rate, the formula becomes simply:

$$N = A/S$$

In Appendix 1 I show how these formulae are derived, and give some examples relating to an actual home in Germany. I also show that there are some cases – in which the discount rate is somewhat higher than the annual fuel price increase - that will never pay back, even if the thermal refit measures last forever[17]. The model also shows that most thermal refit cases can never pay back if the discount rate goes above 10%. Since most people's discount rate is in the 12%-20% range, this puts a question-mark over the German governments' claim that thermal renovation to EnEV standards is generally *wirtschaftlich,* if seen from a realistically social perspective

There are three main differences between this and the official model. Firstly, it gives the result in terms of year to payback, rather than a comparison of costs and benefits per kWh of energy over a fixed period. My interviews with homeowners indicate that they find this the more meaningful measure. Secondly, it puts the onus on the homeowner to make choices as to what to include in the 'additional thermal' costs of the job, how much energy is likely to be saved in his or her particular case, what to include in the discount rate, and what the cost of energy is likely to do over time. These decisions are taken out of the hands of officials and given back to the homeowner. Thirdly, it avoids the peculiarities of averaging exponentials, noted above, and of attaching the discount rate to the costs.

Martinaitis et al. (2007: 193) argue that simple payback time models – of which this is one - cannot distinguish between the economic efficiencies of refits that have different lifetimes. If the payback time of two measures is the same but their lifetimes are different, the two measures will not be equally economically efficient. This is true in theory, and it would matter if we were comparing, say, loft insulation, which has a very long lifetime, with external wall insulation, which is fragile and exposed to the elements. But in this case, a homeowner would do two separate sums and decide whether the payback time for each feature made sense in terms of its possible longevity.

Further, the model has the advantage that, the earlier the payback time, the more economically efficient a refit measure is, regardless of its possible (and unknowable) lifetime. This is because an early payback time means it will move into profit soon, and so bring bigger long-term gains.

However the model has the disadvantage that it does not reveal how economically efficient a refit is in terms of the kWh it saves per euro invested. The fact that a refit pays back early does not necessarily indicate that it is a better investment than alternatives on offer, a point I explore in more detail elsewhere (Galvin, 2010).

6. Discussion and conclusions: Mathematics, values and politics

I have attempted to show that certain values and political stances can be embedded in the mathematics which experts employ to produce knowledge for politicians. This is closely related to the question of the extent to which it is possible for scientific findings to be value-free, but goes further, as it concerns not so much how scientists interpret natural phenomena, but the mathematical modelling used to relate the knowledge of natural phenomena to social needs.

It raises four general issues in the discussion of mathematics, politics and values.

6.1 Mathematics and social elites

The first issue is whether mathematics – the language of certain 'experts' - is a substantive thing that everybody must bow down to, or merely a social construction of a particular elite group. A Foucauldian analysis might suggest that mathematicians are a socially privileged group whose specialist knowledge gives them power over others, much like doctors have power over patients (Torfing, 1999: 90-91) or mental health experts can 'position' people as sane or insane because of their specialised and rather esoteric knowledge as to how human beings are supposed to function (Parker, et al., 1995).

This relates to Wittgenstein's (1978 [1956]) insight that mathematics is a social activity, and that nature does not 'cause' its principles, theorems and axioms to be seen, discovered and believed by mathematicians. Bloor (1973) takes this further, arguing that mathematics is simply a product of culture and not a reflection of 'truth-as-such'. Knowledge is something that human beings produce discursively in their social interactions, and mathematics is one form of knowledge that humans find useful. The reasons we think in a certain way about a mathematical issue can only be explained, Bloor argues, by reference to social factors. If this is the case, mathematicians can be seen as a social group who have attained status and power by developing a profession that requires rites of entry which only they hold the keys to.

The difficulty with Bloor's view is that it ignores the way that humans actually interface with mathematics, namely by doing mental *work.* People have to work hard to construct mathematical symbolism that accords with the way nature tends to function, and to follow that symbolism through consistently. This is why many learners fail to achieve mathematical proficiency: this 'work' requires exacting skills. As Barrow (2010) argues, mathematics is the sum total of all possible patterns, and since the universe functions according to patterns, it is not surprising that mathematics can map it quite well. Whatever symbolic systems an intelligent being might choose to use for the language of mathematics, the patterns it is symbolising run along their own lines, quite independently of our social needs in behaving as mathematicians. This is why, it was possible, for example, for Leibniz and Newton to develop calculus quite independently of each other at the same time, in different parts of Europe, for completely different purposes[18], using different sets of symbols but producing the same 'truths'.

In this respect there is no ground for doubting that the mathematical structures and logic *within* the axioms and theorems that Germany's EnEV modellers choose to use in their models are correct. Once these experts had chosen the parameters and structure of their model and the dimensions of these, they had no choice as to how to perform the calculations. We can assume that whatever culture had taught them mathematics, on whatever planet in the universe, their results would have come out the same.

6.2 Mathematics as a cultural worldview

The second issue is the idea that mathematics is only one way of describing the world, i.e. only one kind of knowledge among many, and a completely idealised one, in that the world is always much more complex and nuanced than the axioms of pure mathematics give credit for. This theme emerges in Latour's (2004) study of the 'politics of nature.' Here the claim is made that presenting nature as an external object, which only 'experts' can understand, acts as a dogma which limits the scope of human action (compare Hajer and Versteeg, 2005: 179). This suggests that mathematics, as a discipline, is *intrinsically* not value-free, as it adopts the position of looking at nature purely in terms of idealised algorithms.

This critique applies to the model discussed here in several ways. Firstly, projects in the real world do not come with hard-and-fast distinctions between 'anyway' costs and 'additional thermal' costs, and any attempt to impose a rigorous division between these is an idealised model that rides roughshod over the nuances and complexities of its real world objects. Secondly, it brackets off aspects of value that are difficult to quantify, such as the loss of an elegant facade when eternal wall insulation is added, the loss of height in a basement when the legal minimum of 16 cm of insulation is attached to the ceiling, or the diminished usefulness of a balcony when its space is intruded upon by thick external wall insulation.

Thirdly, this is related to why the discount rates used in the model make no allowance for opportunity costs or risk. According to Federal officials, it is too difficult to guess at a value for such things, as they are too personal[19]. Yet this is not a good argument for assuming they equate to zero. The choice of zero for a value which is indeterminate but always numerically positive is a skewed political choice and, in this case, one that makes a renovation project appear more likely to be economical.

Hence the very nature of mathematics, as a numeric and algebraic modelling took for a real world, socio-technical project in a diversely varied material ensemble such as the built environment, is problematic by its very nature. A model controlled by the state should not simplify as ruthlessly as this one, if it is to be of general use and the basis for regulations.

6.3 Mathematics and social control

The third issue is the way mathematics is used here to control how people should think about their homes. As I showed above, the model produces a particular type of answer (whether a job pays back over a 25 year period); sets out which parameters are to be included (lifespan, cost of job, fuel price rises, discount rate and reduction in fuel use, but not loss of value due to loss aesthetic appeal or living convenience); and tells how these are to be worked out (by telling us how we have to separate 'anyway' and 'additional thermal' costs, by using building physics only to work out fuel savings, and by slicing risk and opportunity costs off the discount rate).

In this way the model sets up and continually reinforces the lines along which people are expected to think about thermal renovation of their homes. In social constructionist terms, it produces social structures that form the basis of how people come to see the world, in relation to this particular area (Davies and Harré, 1990; Hajer, 1995: 55-58). It can thereby be seen as a kind of power play, in which those who promote the model get their way in the world by producing certain social norms and expectations as to how these expensive jobs should be done. The super-insulation job, spoken of everywhere as *wirtshcaftlich* and the correct way to do things, comes to be seen as 'normal', while alternative approaches are seen as 'deviant' (cf. Foucault, 1976; 1977 [1975], and see Baert and da Silva, 2010: 194ff). Most of the policy actors I interviewed spoke of the conceptualisation of thermal renovation according to the model discussed here as *'richtig'* - meaning 'right', 'true', 'correct', or 'proper'. It was beyond question, part of the mental furniture with which one thinks, rather than something to be thought about critically.

The alternative model proposed in Section 5 above could be seen as an act of resistance against this power-play, as it continually invites homeowners to step out of the *'richtig'* way of thinking and propose their own parameters. In its simplified form (**N = A/S)** it is, in fact, what most of my homeowner interviewees actually use when defending themselves against the dominant claim that super-insulation would be *wirtschaftlich* for them.

6.4 Responsible mathematics

If there is one aspect of the official model that could be seen as irresponsible, it is in the hidden properties of the average of the exponential. Although many people would know that annual fuel price rises lead to an exponential curve for fuel savings, very few would realise that this curve does not reach its average value until after the halfway point. The natural tendency is to think of averages as happening about halfway along a curve. Hence many homeowners would be led to think that their annual returns would draw level with their annualised costs about 2-3 years earlier than they would in fact.

Further, it takes quite sophisticated thinking to realise that up until this time they will be running at a loss every year – and that therefore, from a purely economic point of view, it would make more sense to hold off doing the job until the annualised cost draws level with the actual fuel cost savings. In other words, work out your annualised cost per kWh of energy saved for a 25 year investment, and count the job as *wirtschaftlich* only if this is less than or equal to the *current* fuel price per kWh. This is the *most* economical way to invest - assuming you choose a 25-year framework. It is the way that brings the biggest return on your investment, rather than simply a way of getting your money back within 25 years.

Since mathematicians have specialised knowledge which many people find difficult to understand, it could be argued that they have a social obligation to be overly transparent with their models, especially if these are to be used as a basis for legal requirements. In all my interviews with government officials, building practitioners and the building physics community, I found only one person, an architect writing government-commissioned reports on the economics of thermal refits, who had noticed the exponential average issue. But this person saw it in a positive light, as it prevented people finding an extra reason to avoid thermal renovation. On the other hand, I found few homeowners who understood the way the model worked. One homeowner, an accountant, had modified the model to take account of the actual projected fuel consumption in his own household. Another, a building engineer, had modified the 'anyway' and 'additional thermal' cost distinction to suit his own views. Using these modifications, each had concluded that renovating to EnEV standards would not be economical for them. But even these two had accepted the exponential average operation without question.

6.5 Mathematical models and peer review

The model discussed here would most likely satisfy a peer review based on its internal consistency and its accordance with the laws of physics. But in terms of its purpose, as a link between materiality and social policy, it is found wanting. Funtowicz and Ravetz (1991; 1993) have offered creative suggestions as to how citizens could be involved in the peer review of such models. Building upon this idea, Yearly (2006) found that citizens' comments on air pollution monitoring models in Sheffield effectively amounted to an extended peer review process. Because of the citizens' more detailed, spatially differentiated experience of air pollution, they were able to point to the models' shortcomings. More generally, Callon et al. (2009[2001]) argue that citizens' local knowledge and hands-on experience of certain environmental issues gives them unique information and insight which would enrich the scientific findings of more distant experts. In the case of thermal renovation, the homeowners' intimate knowledge of their buildings and of lifestyle and budgetary issues gives them insights which should not be ignored if a model is to relate physics to budgets, home heating habits, and specific building quirks. In this sense the accepted model is not just mathematically politicised, it is also based on poor science.

In conclusion, we need to look carefully and critically at models derived or used by scientific experts to inform government and other authorities as to how the science of a particular area relates to social or environmental concerns. We need to identify what features of the model are good science and what are bad science, and where the distinctions lie between science on the one hand, and the modellers values and politics on the other. This paper has shown that in doing his, we need to pay special attention to the mathematical structures, both within the model, and on the interface between its algorithms and the way its parameters are both chosen and quantified. We also need to look carefully at how its rather idealised structures relate to the nuances and textures of the world, which are anything but idealised. Mathematics itself may be blandly a-political and value-free in the way its algorithms intermesh, but it becomes political as soon as we attempt to connect it with the things that matter to us in the world[20].

Footnotes

[1] A 'thermal bridge' is an area of the building envelope that is not well insulated compared to its surroundings.

[2] The actual wordings are:

'Die Maßnahme ist wirtschaftlich, wenn die eingesparten Energiekosten höher sind als die Kosten der Energiesparmaßnahme' (Kah and Feist, 2005: 9); and

'Eine Energiesparmaßnahme ist dann als wirtschaftlich anzusehen, wenn die annuitätischen Energiekosteneinsparungen größer sind als die annuitätischen Kosten' (Enseling and Hinz, 2006: 22).

[3] Interview with Ingrid Vogler, Chief Researcher, Association of German Housing providers www.gdw.de).

[4] www.dena.de

[5] In 28 interviews with German policy actors and homeowners all the 'guesses' for the future annual increase in the cost of heating fuel were in the range 5% to 8%.

[6] Federal officials at both the German Energy Agency (DENA) and the Ministry for Housing (BMVBS) confirmed that this was happening in Germany.

[7] Interviews with homeowners in Augsburg.

[8] *Kreditanstalt für Wiederaufbau* (KfW): www.kfw.de

[9] Personal communication with Grosvenor finance manager.

[10] Interview with Dr Georg Vogelsang, Director of BASF housing subsidiary LUWOGE.

[11] Interview with Arne Neuhaus, Technical Manager of *Erbbauverein.*

[12] Available at the Agency's website, www.dena.de

[13] Typically it consists of a heat monitor on each radiator, which measures, through evaporation of fluid in a capsule, how warm the radiator gets, for how long. If a room is warmed by sunlight but the radiator is off, the meter will register consumption of energy and the occupier will be charged for this.

[14] I focus on the version of the model using a 25 year lifespan rather than 20 years plus residual, as this is the simplest to communicate to a wide audience. In any case the latter version brings very similar results.

[15] Most of the modellers do the steps in a different order, which I find unnecessarily contorted and non-transparent. The order I am using is mathematically identical but, I believe, clearer to grasp.

[16] Martinaitis et al (2007) propose a two-factor method for appraising the economics of a thermal renovation, in which the 'anyway' costs are included, but weighted differently from the 'thermal improvement' costs.

[17] . These cases will crash a calculator or spreadsheet (as you cannot have a log of a negative number or of zero) so another way of dealing with them is given in the appendix.

[18] Newton developed calculus to explain his laws of motion. Leibniz did so to expand his somewhat esoteric philosophical reflections.

[19] Interviews with officials of the Federal Ministry for Housing (BMVBS) and the Federal Office for Building and Planning (BBR - *Bundesamt für Bauwesen und Raumordnung).*

[20] I wish to thank Gill Seyfang, Irene Lorenzoni, Per Simmons (University of East Anglia) and jeff Vickers (University of Cambridge) for their helpful suggestions on the initial draft of this paper.

References

Ayer, A.J. (1952) *Language, Truth and Logic (second edition),* News York: Dover Publications.

Barnes, B. (1977) *Interests and the Growth of Knowledge,* London: Routledge and Kegan Paul.

Barrow, John D. (2010) 'Simple Reality: From Simplicity to Complexity – and Back Again', in Bill Bryson (ed.) *Seeing Further: The Story of Science and the Royal Society,* London: Harper Press, pp. 360-383.

Berkhout, P.; Muskens, J. and Velthuijsen, J. (2000), 'Defining the rebound effect.' *Energy Policy* 28: 425–432.

Bhaskar, Roy (1978 [1975]) *A Realist Theory of Science: Second edition*. London: Verso.

Bloor, D. (1973), 'Wittgenstein and Mannheim on the sociology of mathematics.' *Studies in the History and Philosophy of Science* 4:173-191.

Bloor, D. (1999) 'Anti-Latour'. *Studies in the History and Philosophy of Science* 30:81–112.

Boardman, A.; Greenberg, D.; Vining, A. and Weimer, D. (1996), *Cost-Benefit Analysis: Concepts and Practice*, Upper Saddle River, N.J. Prentice Hall.

Callon, Michel; Lascoumes, Pierre and Barthe, Yannick (2009) *Acting in an Uncertain World: An Essay on Technical Democracy,* (original French edition 2001) Cambridge (Mass) and London: MIT Press.

Davies, B. and Harré, R. (1990) 'Positioning: The Discursive Production of Selves,' *Journal for the Theory of Social Behaviour,* 20(1): 43-63.

Deutsche Poroton (2009) *Wenn der Specht die Dämmung kostet,* Presseinformation-Nr. 09/2009 www.poroton.de

Dryzek, John S. and Dunleavy, Patrick (2009) *Theories of the Democratic State,* Basingstoke: Palgrave Macmillan.

Enseling, Andreas and Hinz, Eberhard (2006) *Energetische Gebäudesanierung und Wirtschaftlichkeit – Eine Untersuchung am Beispiel des 'Brunckviertels' in Ludwigshafen,* Darmstadt, Institut Wohnen und Umselt GmbH.

Feist, W. (1997) *Überprüfung der bedingten energetischen Anforderungen im Gebäudebestand bei Beibehaltung der gegenwärtigen Rechtsgrundlage der Wärmeschutzverordnung: Studie im Auftrag des BMBau,* Passivhaus Instittut, Darmstadt.

Feist, W. (1998) *Wirtschaftlichkeitsuntersuchung ausgewählter Energiesparmaßnahmen im Gebäudebestand, Fachinformation PHI-1998/3,* Passivhaus Institut, Darmstadt.

Fischer, Frank (2003) *Reframing Public Policy: Discursive Politics and Deliberative Practice,* Oxford: Oxford University Press.

Foucault, Michel (1977 [1975]) *Discipline and Punish: The Birth of the prison* (trans. A. Sheridan), New York: Pantheon.

Foucault, Michel (1976) *The History of Sexuality,* Vol. I, 1981 trans., Harmondsworth: Penguin

Friedrich, Malte; Becker, Daniela; Grondy, Andreas, Laskosky, Francisca; Erhorn, Hans; Erhon-Kluttig; Hauser, Gerd; Sager, Christina and Weber, Hannah (2007b) *CO2-Gebäudereport 2007 (CO2-Building Report 2007 Full Report),*
im Auftrag des Bundesministeriums für Verkehr, Bau und Stadt-entwicklung (BMVBS) (Commissioned by the Ministry of Transport, Buildings and Urban Development), Stuttgart: Fraunhofer Institut Bauphysik.

Funtowicz, S O. and Ravetz, J.R. (1991) 'A new scientific methodology for global environmental issues'', in R. Costanza (ed.), *Ecological Economics,* New York: Columbia University Press, 137-152.

Funtowicz, S. O. and Ravetz, J. R. (1993) 'Science for the post-normal age,' *Futures,* 25:740 –755.

Galvin, Ray (2010) 'Thermal upgrades of existing homes in Germany: The building code, subsidies, and economic efficiency,' *Energy and Buildings*, 42(6): 834-844.

Greening, L.; Greene, D. and Difiglio, Carmen (2000), 'Energy efficiency and consumption – the rebound effect – a survey.' *Energy Policy* 28(6/7): 389–401.

Hajer, Maartin (1995) *The Politics of Environmental Discourse: Ecological Modernisation and the Policy Process,* Oxford: Clarendon.

Hajer, Maartin and Versteeg, Wytske (2005) 'A Decade of Discourse Analysis of Environmental Politics: Achievements, Challenges, Perspectives,' *Journal of Environmental Policy and Planning,* 7(3): 175-184.

Hajer, Maartin and Wagenaar, Hendrik (eds) (2003), *Deliberative Policy Analysis: Understanding Governance in the Network Society,* Cambridge: Cambridge University Press.

Handwerk (2008) *Spechte hacken faustgroße Löcher in Wärmedämmplatten. Warum? Vogelkundler haben eine einfache Erklärung parat.*
http://www.handwerk.com/tock-tock-tock/150/42/26461/ 14.10.2008

Hartle, Douglas G. (1976) 'Techniques and Processes of Administration,' *Canadian Public Administration,* 19: 21-33.

Healy, Patsy, De Magalhaes, Claudio, Madanipour, Ali and Pendlebury, John (2003) 'Place, identity and Local Politics: Analysing Initiatives in Local Governance,' in Maarten Hajer and Hendrik Wagenaar (eds.) *Deliberative Policy Analysis: Understanding Governance in the Network Society,* Cambridge: Cambridge University Press.

HM Treasury (2010) *The Green Book: Appraisal and Evaluation in Central Government,* London: TSO. Available online at http://www.hm-treasury.gov.uk/d/green_book_complete.pdf Accessed 29 March, 2010.

Kah, Oliver and Feist, Wolfgang (2005) *Wirtschaftlichkeit von Wärmedämm-Maßnahmen im Gebäudebestand 2005,* Darmstadt: Passivhaus Institut.

Lasswell, H.D. (1951) 'The Policy Orientation,' in H.D. Lasswell and D. Lerner (eds), *The Policy Sciences,* Stanford, Calif: Stanford University Press, pp3-15.

Latour, Bruno (1987) *Science in Action: How to Follow Scientists and Engineers Through Society*. Cambridge, MA: Harvard University Press.

Latour, Bruno (2004) *Politics of Nature: How to Bring the Sciences into Democracy,* Cambridge, MA: Haarvard University Press.

Latour, Bruno (2005) *Reassembling the Social: An Introduction to Actor-Network Theory*. Oxford and New York: Oxford University Press.

Lynn, E. Lawrence Jr. (1999) 'A Place at the Table: Policy Analysis, its Postpositiv Critics, and the Future o Practice,' *Journal of Policy Analysis and Management,* 18(3): 411-424.

Martinaitis, Vytautas; Kazakevičiusb, Eduardas; and Vitkauskasb, Aloyzas (2007) 'A two-factor method for appraising building renovation and energy efficiency improvement projects', *Energy Policy* 35: 192–201.

Martinaitis, Vytautas; Rogoža, A; Bikmaniené, I. (2004) 'Criterion to Evaluate the "Twofold Benefit" of the Renovation of Buildings and their Elements,' *Energy and Buildings,* 36(1): 3-8.

Parker, I., Georgaca, E.; Harper, D.; McLaughlin, T.; and Stowell-Smith, M. (1996) *Deconstructing Psychopathology.* London: Sage.

Pecht, Marco (2009) `Immobilien: Wenn der Specht am Passivhaus nascht', *Berliner Zeitung,* 11. April, 2009: Bauen.

Pickering, Andrew (1984) *Constructing Quarks: A Sociological History of Particle Physics,* Edinburgh: Edinburgh University Press.

Sabatier, Paul and Jenkins-Smith, Hank (eds.) (1993) *Policy Change and Learning: An Advocacy Coalition Approach,* Oxford: Westford.

Schuler, Andreas; Weber, Christoph and Fahl, Ulrich (2000), 'Energy consumption for space heating of West-German households: empirical evidence, scenario projections and policy implications,' *Energy Policy* 28: 877-894.
Torgerson, Douglas (1986) 'Between Knowledge and Politics: The Three faces of Policy Analysis,' *Policy Sciences* 19:33-59.

Sorrell S and Dimitropoulos S, (2008), 'The rebound effect: microeconomic definitions, limitations and extensions,' *Ecological Economics* 65(3): 636-649.

Torfing, Jacob (1999) *New Theories of Discourse: Laclau, Mouffe and Žižek,* Oxford: Blackwell Publishers.

Torgerson, Douglas (1986) ‘Between Knowledge and Politics: The Three faces of Policy Analysis,’ *Policy Sciences* 19:33-59.

Wittgenstein, Ludwig, 1978 [1956], *Remarks on the Foundations of Mathematics*, Revised Edition, Oxford: Basil Blackwell, G.H. von Wright, R. Rhees and G.E.M. Anscombe (eds.); translated by G.E.M Anscombe.

Yearly, Steven (2006) ‘Bridging the science - policy divide in urban air-quality management: evaluating ways to make models more robust through public engagement,’ *Environment and Planning C: Government and Policy,* 24: 701-714.

Appendix 1. Deriving Formulae for Thermal Refit Payback Time Calculations

Let:

E = percentage annual increase in heating energy price

D = percentage annual discount rate

F = combined annual fuel increase and discount rate factor (see below)

S = annual saving in fuel costs due to refit, in present value €s.

A = sum of value of energy savings over n years, in present value €s

In this method we work out the *present value* of energy saved in future years.

F is a factor that converts a future year’s monetary saving to present value.

$F = (1 + E/100) / (1 + D/100)$

In any particular year after the initial year, i.e. year n +1, we multiply the monetary saving by f^n to convert it to present value.

The sum of all the annual savings over n years is:

$A = S + S.F + S.F^2 + S.F^3 + ... + S.F^{n-1})$

$= S. (1 + F + F^2 + F^3 + ... + F^{n-1})$ ....(1)

Multipling both sides by F:

Hence: $A.F = S. (F + F^{2} + F^{3} + F^{4} + ... + F^{n})$ ....(2)

Subtracting equation (1) from equation (2):

⇨ $A F – A = S. (F^{n} – 1)$

⇨ $A. (F – 1) = S. (F^{n} – 1)$

⇨ $A = S. (F^{n} – 1) / (F – 1)$ ....(3)

In the year the project achieves payback, the total monetary gain, in present value terms, is equal to the cost of the thermal refit. Hence we can substitute this cost for A in equation (3) and solve it for n.

Hence : $(A/S) \times (F – 1) = (F^{n} – 1)$

➔ $(A/S) \times (F – 1) + 1 = F^{n}$

➔ **$n = \log [(A/S) \times (F-1)+ 1] / \log F$ ....(4)**

Example: (an actual example from empirical research)

A 3-storey house in a Bavarian village already has 3cm of polystyrene external wall insulation and a render in good condition. The windows were replaced in the late 1980s and the boiler was replaced with a modern, efficient model in 2004. An energy advisor has told the owners that if they re-insulate the external walls to a thickness of 12 cm this will cost €40,000 and they will reduce their heating energy consumption by 30%. This amounts to only 34 kWh/m^{2}a, as the 2 occupants seldom heat the upper storeys above a recommended minimum. The current annual heating fuel bill is 1700 euros and the energy price 0.06 euros per kWh. The floor area is 250 m^2. Hence the expected monetary saving per year, in present value terms, is:

$S = 34\ kWh/m^2a \times 250\ m^2 \times €0.06 = €510$ (or simply take 30% of €1700)

Using an expected annual fuel price rise of 7% and a discount rate of 5% we get;

$F = (1 + 7/100) / (1 + 5/100)$

$= 1.019$

To find the number of years to payback, we use

n = log [(A/A) x (F-1)+ 1] / log F

= log [(40,000/510) x (1.019-1)+ 1] / log 1,019

= 48 years

However, the expected lifetime of the refit is much less than this. Hence this investment could not be described as *wirtschaftlich* (economic).

This example assumes that the render is not due for replacement, as this was the actual situation in the case studied. If the render did have to be replaced ‚anyway' this would reduce the cost of the 'additional thermal' costs to €21,000. In this case, the calculation is:

n = log [(21000/510) x (1.019-1)+ 1] / log 1.019

= 31 years

Again, it is clear that the investment will not pay back during the lifetime of the renovations. This is largely because the house is already relatively energy efficient due to its solar and modern heating system, its loft insulation and its relatively modern windows.

Thermal refits that can never pay back

Consider the equation for the payback time of a thermal refit:

n = log [(A/S) x (F-1)+ 1] / log F .... (4)

If (A/S) x (F-1)+ 1 <= 0, then the equation has no solution, since there cannot be a logarithm of a negative number or of zero.

This is the case where F <= 1 – S/A

This represents a case where the annual fuel price rise, E, is smaller than the discount rate, D, so that each year's savings is smaller than that of the previous year, *and* the annual saving in present value terms, S, is quite small in relation to the cost of the refit, A.

This can also be seen in equation (1), namely:

A = S. (1 + F + F^2 + F^3 + ... + F^{n-1}) ....(1)

If F < 1, this will be a diminishing series. Payback will be reached when

S/A (1 + F + F^2 + F^3 + ... + F^{n-1}) = 1

However if its sum to infinity is less than 1, payback can never be achieved. This is why equation (4) has no solution for this case.