Integrating the rebound effect: a more precise, coherent and useful predictor of gains and losses through energy-efficiency upgrades in home heating

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Abstract

Although the term 'rebound effect' is often used very loosely, one particular definition has won acceptance for its conceptual clarity and mathematical robustness: the energy efficiency elasticity of demand for energy services. This is formulated as a partial differential, and its structure enables transformations with price and energy elasticities. However, when considering heating energy efficiency upgrades of homes, the mathematics breaks down because these upgrades involve large changes in efficiency, energy and energy services, whereas differential calculus only holds true for very small changes. This could be one reason why existing estimates of rebound effects are so diverse. This paper shows how this limitation can be remedied, using the German housing stock as a case study. A curve of consumption/efficiency for this stock is derived from empirical studies and, based on the mathematical definition of the rebound effect, a rebound effect relation is derived from this. This is then integrated over the likely ranges of energy efficiency upgrades that would correspond to the government's policy of reducing consumption by 80%. The model is found to be mathematically coherent, and suggests energy service rebounds of 28-39% for the German stock as a whole if the 80% goal is achieved.

Keywords:

Rebound effect; German housing stock; energy efficiency upgrades; thermal retrofits; energy efficiency elasticity

1. Introduction

This paper offers a methodology for finding averaged rebound effects for energy-efficiency upgrades in large samples of existing homes. It shows how the mathematical structure of the rebound effect, as commonly defined in econometric literature, can be extended so as to give coherent results for energy and energy services rebound effects through such upgrades. This is not possible without this extension, as these upgrades involve very large increases in energy efficiency, whereas this formulation of the rebound effect is based on differential calculus, which holds true only for very small changes.

The term 'rebound effect' is used in a variety of ways, to represent the shortfall in energy savings gains or the increase in energy services consumption that often follow an improvement in energy efficiency. Much of the literature on this phenomenon is imprecise in its definition of the 'rebound effect' (e.g. Bonino et al., 2012; Deurinck et al., 2012; Jakob et al., 2012; Hinnells, 2008). Results of different studies in the same field are often difficult to compare, because different metrics are being used. For example in investigating rebound effects for home heating upgrade**s**, various studies compare: shortfalls in energy savings with expected energy savings (e.g. Haas and Biermayer, 2000; Guerra Santin et al., 2009; Jin, 2007); actual consumption with predicted consumption (e.g. Demanuele et al, 2010; Tronchin and Fabbri, 2007); percentage reduction in energy consumption with percentage increase in energy efficiency (e.g.

Madlener and Hauertmann, 2011); increase in indoor temperature with percentage increase in energy efficiency (e.g. Milne and Boardman, 2000); or increases in other formulations of energy services consumption, such as comfort, with increases in energy efficiency (e.g. Howden-Chapman et al., 2009).

Despite these differences, however, one particular formulation of the 'rebound effect' has come to dominate economics-oriented literature, and this formulation has a structure which suggests a quest for high precision. This may be described as the *energy efficiency elasticity of demand for energy services*, or more loosely, the infinitesimal proportionate change in the take of energy services as a fraction of an infinitesimal proportionate change in energy efficiency. Expressed formally this is:

$$R_\varepsilon(S) = \frac{\partial S}{S} / \frac{\partial \varepsilon}{\varepsilon}$$
$$= \frac{\partial S}{\partial \varepsilon} \cdot \frac{\varepsilon}{S}$$

where S represents the quantity of energy services consumed (e.g. warmth in the home; kilometres travelled; hours using a washing machine); and ε represents the efficiency with which energy is converted into those services. This definition is usually traced to Berkhout et al. (2000), though its expression as a precise mathematical formulation was first offered by Sorrell and Dimitropoulos (2008). It has since become something of a standard definition, with many authors at least noting its mathematical basis, even if they then go on to use a different definition (e.g. Druckman et al., 2011)

There are two reasons this is structured as a partial differential. Firstly, it allows for the possibility that changes in energy services consumption may come about through more than one influence at the same time. For example $\frac{\partial S}{\partial \varepsilon}$ may be accompanied by a change in the exogenous price of energy, a shift from colder to warmer winters, or a rearrangement of room use in a home. This paper will not consider the effects of changes in these variables, though it should be possible to incorporate them into an overall value for the rebound effect.

However, if these variables are controlled for there is no reason why $\frac{\partial S}{\partial \varepsilon}$ may not be treated as a normal differential, $\frac{dS}{d\varepsilon}$.

The second reason this is structured as a (partial) differential is that it is designed, as a definition, to deal only with infinitesimal changes in energy service take S in response to infinitesimal changes in energy efficiency ε. This has two great advantages. One is that it allows for the possibility that S as a function of ε is *non-linear.* In such a case, the rebound effect will be different at each point along the curve of services level against energy-efficiency, and this can be captured by the differential of the services/efficiency function $S(\varepsilon)$, i.e. ${\partial S}/_{\partial \varepsilon}$, multiplied by ${\varepsilon}/_{S}$. As we shall see, this makes it a very useful mathematical structure for dealing with large sample averages of the effects of energy efficiency upgrades on homes, where smooth, differentiable curves of $S(\varepsilon)$ are obtainable.

The second advantage is that it enables transformations to be made with functions for price elasticity of demand for energy services, which are usually also non-linear and based on large sample averages (see discussion in Sorrell, 2007; Sorrell and Dimitropoulos, 2008; Sorrell et al., 2009).

When used mathematically 'correctly', i.e. for dealing with infinitesimal changes, there is a perfect coherence between two different forms of the rebound effect, namely:

$R_\varepsilon(E) - R_\varepsilon(S) = -1$, where $R_\varepsilon(E) = \frac{\partial E}{\partial \varepsilon} \cdot \frac{\varepsilon}{E}$ and E = energy consumption.

Alternatively this can be expressed as $R_\varepsilon S - R_\varepsilon(E) = 1$. This means that, for an infinitesimal increase in energy efficiency, a certain proportion of the efficiency increase goes to increase the take of services, while the other proportion goes to reduce the energy consumption. The two proportions add up to 1.0 precisely, i.e. *all* the energy efficiency increase is accounted for.

However, the very advantages of this precisely defined formulation of the rebound effect afford it big disadvantages for dealing with domestic heating energy efficiency upgrades. These upgrades produce very *large* changes in energy efficiency, often over 150%, which violates the rule of infinitesimal changes (a 60% reduction in energy demand is mathematically equivalent to a 150% increase in energy efficiency). Indeed, even for changes of as little as 10% the results begin to become incoherent. As will be shown below, in such cases the two proportions of the energy efficiency gain do *not* add up to 1.0. Something is lost; not all the energy efficiency gain is accounted for.

This paper offers a way of moving the mathematical structure of this 'classical' formulation of the rebound effect one step forward, so that this incoherence is overcome. While it could be argued that this is a purely theoretical exercise, of little practical use, there are two reasons it is important. Firstly, the formulation of the rebound effect outlined above has now become very influential in economics and much policy literature. If it is used, it should be used correctly. The vagueness and imprecision of results in many studies (see, e.g., summaries in Yun et al., 2013) may not all be due to difficulties in obtaining precise empirical data, but also due to wrong use of the formulae. Even if there are large spreads and uncertainties in the empirical data, this is not an argument for processing this data with imprecise or wrongly conceived mathematics.

Secondly, higher precision of empirical data is possible when dealing with averages of large datasets. Many European countries are committed to reducing the heating energy consumption of their housing stocks by 80% by 2050. We may wish to see what magnitude of rebound effects can be expected from the energy-efficiency upgrades that will be required for such a transformation. What proportion of the *average* energy-efficiency upgrade will be taken, on average, as an increase in energy services take, and what proportion will go to reducing energy consumption, if we attempt this 80% reduction? As will be shown below, there is enough data on the average heating energy consumption of German households at each specific energy performance rating (EPR) value, for us to be able to make a first attempt to map the consumption curves that provide the algebraic functions necessary for the rebound effect formulations to be used with more precision. However, in their present form, these formulations are only good for infinitesimal changes, not for the very large energy-efficiency changes that correspond with an 80% reduction in the EPR.

This paper therefore offers a way of moving the mathematics beyond this impasse. It shows how the rebound effects $R_\varepsilon(E)$ and $R_\varepsilon(S)$ can be expressed as functions of the EPR, and how these can be integrated over the full range of the consumption curve, to produce mathematically coherent rebound effect values for any magnitude of energy-efficiency upgrade, from any starting point on the EPR scale. It does this using the heating energy consumption of the German housing stock as a model, since there is sufficient empirical data available in this sphere for credible modelling to be achieved.

This is not to claim that these rebound effects *will* occur, nor that the model of the heating energy consumption of the housing stock is 100% accurate. What it does show, however, with 100% precision and internal mathematical coherence, is the values of the rebound effects that correspond to this data so modelled. Most important, it offers a method that takes the mathematics of the rebound effect a further step forward, so that its definition in terms of infinitesimal changes can become an asset rather than something to be brushed over.

Section 2 of this paper derives a model of the heating consumption of the German housing stock, and uses it to illustrate the incoherence of applying the rebound effect, as described above, to large increases in heating energy efficiency. Section 3 works through a series of mathematical steps to produce definite integrals of the rebound effect $R_\varepsilon(E)$ and $R_\varepsilon(S)$, which turn out to be infinite series of terms of ever-decreasing magnitude. Section 4 shows how these series are evaluated, using a basic computer program, over any span of energy-efficiency upgrade. In Section 5 the results are discussed, and Section 6 concludes.

For clarity of understanding, the following terminology will be used in relation to energy consumption in domestic buildings:

The **'demand'** or **'calculated demand'** is the theoretical heating energy consumption which would provide 100% of the energy services necessary for full thermal comfort in the home. It is equivalent to the EPR and is represented by the variable D.

The **'consumption'** or **'energy consumption'** is the measured or metered heating energy consumption of the current occupants of the dwelling. It is represented by the variable E.

Heating energy consumption figures, for both demand and actual consumption, are given in kilowatt-hours per square meter of useful living area per year **(kWh/m²a)**.

The term **'heating'** refers to space and water heating combined, though the findings of this study are applicable to either of these separately or both together.

2. A model for heating energy consumption

2.1 Finding a consumption/demand curve

In a study of 36,000 dwellings the German Energy Agency *(Deutsche Energie-Agentur* - DENA) finds the average measured heating energy consumption in the German housing stock to be 30% below the average calculated demand (DENA, 2012: 42-43). Sunikka-Blank and Galvin (2012) brought together existing datasets of over 3,700 German dwellings and found a consistent form of relation between specific demand values and the average consumption at each demand value. For any particular value of demand, the average consumption of dwellings with that demand figure followed a mathematically predictable pattern.

For example, in Jagnow and Wolf (2008), in a study of detached houses and small and large apartments in small and large buildings, with heating systems employing gas, oil and district heating (n=200) the relation could be mapped as:

$$E = 12D^{0.48} - 20$$

Using data from a national survey (n=1,702), Loga et al. (2011) plotted consumption and demand for residential buildings of all types with fewer than 8 dwellings, and mapped the relation as:

$$E = \frac{1.3D}{1 + \frac{D}{500}} - 0.2D : \quad 100 < D \leq 500$$

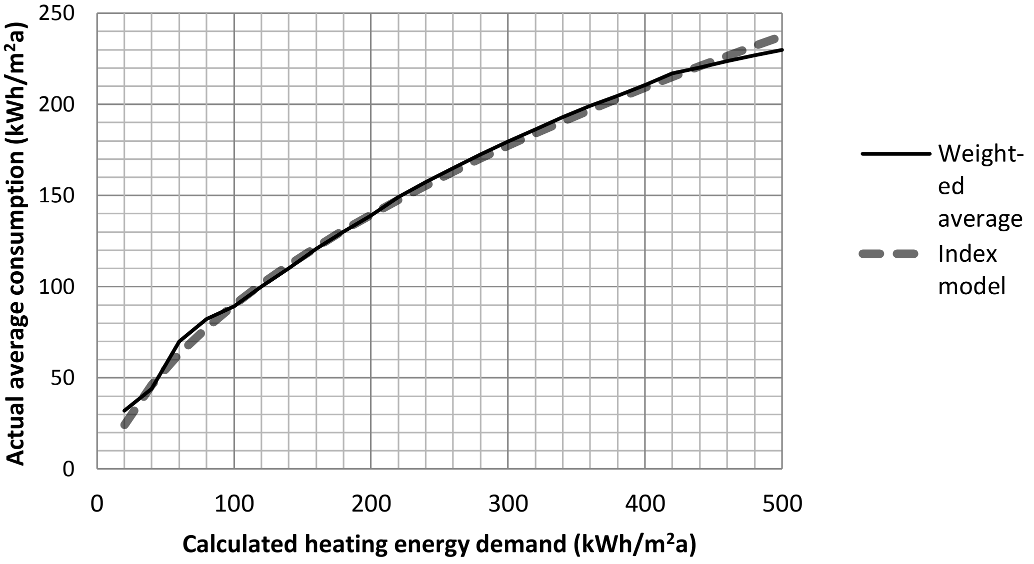
This can be approximated very closely by the relation:

$$E = 14D^{0.48} - 8$$

Data from these and other sources (Erhorn, 2007; Kaßner et al., 2010; Knissel and Loga, 2006) were examined and the average value of E was calculated for each specific value of D between 20 and 500kWh/m²a, weighted according to the number of data points in each dataset over its respective range. Trial and error curve fitting resulted in a relation that mapped very closely to this weighted average, namely:

$$E = 12D^{0.499} - 29.3 \qquad (1)$$

Figure 1. Curve fitting of index model to weighted average of heating consumption/demand for German housing stock



This is displayed in Figure 1. The error between the weighted average (solid line) and the fitted curve (broken line, labelled 'index model' on the legend) averages 1.6kWh/m²a over the range 100 < D < 400, but increases to 7.8kWh/m²a for the lowest value of D, i.e. 20kWh/m²a, and increases again for D > 440.

We are not suggesting this is the *definitive* mapping of average consumption for each demand value. While most of the data were from randomly selected homes, some of the studies of smaller datasets did not make their selection method clear. Further, the curve fitting follows an idealised mathematical form, and it is possible that the wobbles in the weighted average line give a more accurate rendering. Nevertheless, the idealised curve is offered as a first attempt at modelling the average heating consumption for each specific value of demand, for at least the range 40 < D < 440.

2.2 Efficiency and energy services

To use the rebound effect formulae we need first to interpret these parameters in terms of energy services and energy efficiency. In Germany the energy performance rating (EPR), here expressed with the variable *D,* is the quantity of heating energy that is needed to make a specific building fully comfortable all year round. The methodology for calculating this quantity is given in a publication of the German Institute of Standards *(Deutsche Institut der Normung* - DIN) numbered DIN-4180 (see DIN, 2003). It takes account of the thermal properties of the building envelope, the heating system, the orientation to the sun, the heating degree days, and the expected 'free' heating from appliances and indoor human activities. According to this standard, to be fully comfortable – i.e. to have 100% energy services – a dwelling must have an indoor temperature of 19°C throughout the whole dwelling and a ventilation rate of 0.7 times the volume of the dwelling per hour. The energy consumption required to achieve this is the EPR, i.e. *D.*

Whether this is a good definition of 100% energy services is beyond the scope of this paper. The issue is further discussed in other DIN publications, namely DIN 33 403, DIN EN ISO 7730 and DIN 1946. But for this paper we will assume that D is the quantity of energy that would be required to provide 100% energy services.

Following Giraudet et al. (2012), and Tighelaar and Menkveld (2011) we make a further assumption, that the level of energy services actually being received is the ratio of actual consumption E to demand D. For example, a household consuming 110kWh/m^{2}a in a dwelling with an EPR of 220kWh/m^{2}a is receiving services S of 0.5, or 50%. This assumption, too, has its weaknesses, since energy services for home heating may well be seen quite differently by different consumers (some like it hot, some like it cool; some like a breeze, others hate a draft; some like warm air, others like cold air and a hot stove, etc.). However, for want of a fully comprehensive, quantifiable definition of energy services, we will use this one, bearing in mind that there is much scope for socio-technical oriented research to improve upon it.

The definition of energy efficiency ε follows from the above. The energy efficiency is inversely proportional to the energy demand, i.e. $\varepsilon = k/D$. But since we will only ever use ε in the form $\epsilon/\partial\varepsilon$, we can drop the constant k and assume that a dwelling that requires 1kWh/m^{2}a to provide 100% energy services is 100% efficient, i.e. for such a dwelling $\varepsilon = 1$. Given these assumptions we also note that $S = \varepsilon \cdot E$.

These assumptions and relations also accord with the reasoning of Sorrell and Dimitropoulos (2008) in their formal, mathematical definitions of the rebound effect. We use them in the analysis that follows.

2.3 The incoherence of the rebound effect for large efficiency gains

According to the German Energy Agency the average heating energy demand in German dwellings is around 220kWh/m^{2}a (DENA, 2012). Consider a thermal upgrade that produces an 80% reduction in heating energy demand in an 'average' dwelling. Using equation (1) above, this a gives point-to-point change in heating energy efficiency in which:

$D_1 = 220$ => $E_1 = 147.43$; $D_2 = 44$ => $E_2 = 50.00$

Applying the above rules to these figures for S and E gives:

$$S_1 = \frac{147.73}{220} = 0.6715; \quad S_2 = \frac{50.00}{44} = 1.1364$$

$$\varepsilon_1 = \frac{1}{D_1} = \frac{1}{220} = 0.00455; \quad \varepsilon_2 = \frac{1}{D_2} = \frac{1}{44} = 0.0227$$

Consider first the energy services rebound effect $R_\varepsilon(S)$

$$R_\varepsilon(S) = \frac{\Delta S}{S} \cdot \frac{\varepsilon}{\Delta\varepsilon} = \frac{1.1364 - 0.6715}{0.6715} \cdot \frac{0.00455}{0.0227 - 0.00455} = 0.1736$$

This implies that 17.36% of the energy efficiency upgrade has gone to increasing the take of energy services, so we would expect that the remaining 82.64% has gone to reducing energy consumption, so as to satisfy the relation $R_\varepsilon(E) - R_\varepsilon(S) = -1$.

However, when we consider $R_\varepsilon(E)$ we find:

$$R_\varepsilon(E) = \frac{\Delta E}{E} \cdot \frac{\varepsilon}{\Delta\varepsilon} = \frac{50 - 147.73}{147.73} \cdot \frac{0.00455}{0.0227 - 0.00455} = -0.1658$$

This implies that only 16.58% of the energy efficiency increase has gone to reduce energy consumption, and that the remaining 83.42% should have gone to increasing the take of energy services.

The discrepancy between these is enormous:

$R_\varepsilon(E) - R_\varepsilon(S) = -0.1658 - 0.1736 = -0.3394$, which is a long way from -1. A full 66.06% of the energy efficiency increase has simply disappeared. In terms of the classic rebound effect definition these results are incoherent and impossible to interpret. This is because the methodology violates the mathematics of that definition: the changes that come through a typical energy efficiency upgrade are large, not infinitesimally small.

We could, of course, dispense with the $\varepsilon/\Delta\varepsilon$ term altogether and simply say that energy services have increased by (1.1364 - 0.6715)/0.6715 = 69% while energy consumption has reduced by (147.73 – 50.0)/147.73 = 66%. But this does not tell us the 'rebound effect', as it leaves out the essential parameter of the increase in *energy efficiency* as the *driver* of the changes in energy services and energy consumption.

Clearly we need a more sophisticated set of mathematical tools if we are to compute the rebound effect, a partial differential, in a large one-off change, and obtain coherent results. In the following section we address this issue.

3. The rebound effect as a definite integral

3.1 Method

Our method involves four steps, in addition to having derived equation (1) above from empirical sources. Firstly, we need equations in ε for the change in services S and for the change in consumption E along the range of likely changes in ε due to an energy efficiency upgrade. We need $S = f(\varepsilon)$ and $E = g(\varepsilon)$.

Secondly, these need to be differentiated and the results multiplied by ε/S and ε/E respectively, to give curvilinear relations for $R_\varepsilon(S)$ and $R_\varepsilon(E)$.

Thirdly, we need $R_\varepsilon(E)$ and $R_\varepsilon(S)$ to be transformed into functions in D, the energy demand. This will give the rebound effect *at every point* along the demand curve. For every value of D along the consumption/demand curve it will tell us what portion of an (infinitesimal) energy efficiency increase will go to increasing energy services, and what portion will go to reducing energy consumption.

Fourthly, we need to integrate the functions in D we have obtained for $R_\varepsilon(E)$ and $R_\varepsilon(S)$, so as to find the definite integrals for each of these, for the range over which D changes due to an energy efficiency upgrade. Dividing the definite integrals by the range of change in D will give the true averages for $R_\varepsilon(E)$ and $R_\varepsilon(S)$ over this range. If our mathematics is correct, then the relation $R_\varepsilon(E) - R_\varepsilon(S) = -1$ will hold true for our numerical results, and our results will be coherent.

3.2 Steps prior to the integration

Consider equation (1), $E = 12D^{0.499} - 29.3$. Substituting $\varepsilon = 1/D$ in this gives:

$$E = 12\varepsilon^{-0.499} - 29.3 \qquad (2)$$

Hence:

$$R_\varepsilon(E) = \frac{\partial E}{\partial \varepsilon} \cdot \frac{\varepsilon}{E} = \frac{-5.988\varepsilon^{-0.499}}{12\varepsilon^{-0.499} - 29.3} \qquad (3)$$

Substituting $D = 1/\varepsilon$ in this gives an equation for the energy rebound effect as a function of *D:*

$$R_\varepsilon(E) = \frac{-5.988D^{0.499}}{12D^{0.499} - 29.3} \qquad (4)$$

This is the relation that needs to be integrated. Before doing so we note that it can be simplified to:

$$R_\varepsilon(E) = (4.983D^{-0.499} - 2.004)^{-1} \qquad (5)$$

Or more generally:

$$R_\varepsilon(E) = (aD^b - c)^{-1} \qquad (6)$$

We also note the general form of the energy consumption/demand relation, equation (1), namely:

$$E = P \cdot D^Q - T \qquad (7)$$

Hence in equation (6):

$$a = \frac{T}{P \cdot Q} \quad b = -Q \quad c = \frac{-1}{Q} \qquad (8, 9, 10)$$

These transformations will enable us to use the integration and programming method described below to work out integrated rebound effects for any heating consumption/demand curve of the form $E = P \cdot D^Q - T$. In Section 2.1 we saw that curves of this form fit well with all the existing datasets. If fuller datasets come to light in future, the method described below will work provided the relation between demand and the average heating consumption for each demand value can be represented by this general mathematical form. If not, the new form will have to be processed by the methodology above according to its own rules of differentiation, and the methodology below according to its rules of integration.

3.3 The integration

The energy rebound effect curve $R_\varepsilon(E)$ has the form

$$R_\varepsilon(E) = (ax^b + c)^{-1}dx$$

Note that we use *x* here rather than *D* as it makes the expressions visually easier to follow through the steps of integration in Appendix 1.

Let $I = \int (ax^b + c)^{-1}dx \;\; where\ x =$ calculated heating demand

Using: $\int udv = uv - \int vdu$

and first setting $u = (ax^b + c)^{-1}$ and $dv = 1 \cdot dx$

we integrate successively by parts, as detailed in Appendix 1. The result is an infinite series:

$$I = x(ax^b + c)^{-1} + \frac{abx^{b-1}}{b+1} \cdot \left(ax^b + c\right)^{-2} + \frac{2a^2b^2x^{2b+1}}{(b+1)(2b+1)} \cdot \left(ax^b + c\right)^{-3} + \frac{6a^3b^3x^{3b+1}}{(b+1)(2b+1)(3b+1)} \cdot \left(ax^b + c\right)^{-4} \;\; \ldots..etc. \qquad (11)$$

This can alternatively be expressed as:

$$I = \sum_{n=1}^{\infty} \frac{(n-1)!\,(ab)^{n-1}x^{(n-1)b+1}(ax^b + c)^{-n}}{[nb+1][(n-1)b+1][(n-2)b+1]\ldots[0 \times b + 1]} \qquad (12)$$

Although this is an infinite series, it will be evident that the magnitude of the terms reduces rapidly for the values of a, b and c that can be encountered in the equation for $R\varepsilon(E)$. In calculating *I* for various values of these variables and for high values of demand, terms beyond the 30th, and in some cases the 15th, were smaller than could be registered by a computer. This increased to 170 terms for demands lower than 28kWh/m^{2}a, but the series was always a diminishing one.

The algorithm (12) was coded as a computer program in Visual Basic, as in this language code can easily be added to produce Excel spreadsheets with printouts of results, from which graphical displays can be produced. The program calculated the definite integrals for values of D (=x) corresponding to pre-and post- energy efficiency upgrade values of the demand. It subtracted the latter from the former and divided the result by the difference between the two values of D to give the average value of $R\varepsilon(E)$ over the span of the upgrade. Expressed formally this is:

$$R_\varepsilon(E)\big|_{D_2}^{D_1} = \frac{I\big|_{D_2}^{D_1}}{D_1 - D_2}$$

3.4 Proving the results are coherent

The results may be deemed coherent if and only if $R\varepsilon(E) - R\varepsilon(S) = -1$ in all cases. The expression for $R\varepsilon(S)$ as a function of D was found for the case where equation (2) holds, i.e. beginning with:

$$E = 12\varepsilon^{-0.499} - 29.3$$

Substituting $S = \varepsilon \cdot E$ gives:

$$S = 12\varepsilon^{0.501} - 29.3 \qquad (13)$$

This gives the energy services rebound effect:

$$R_\varepsilon(S) = \frac{\partial S}{\partial \varepsilon} \cdot \frac{\varepsilon}{S} = \frac{6.012\varepsilon^{0.501} - 29.3\varepsilon}{12\varepsilon^{0.501} - 29.3\varepsilon}$$

Substituting ε= $1/D$ gives:

$$R_\varepsilon(S) = \frac{6.012D^{0.499} - 29.3}{12D^{0.499} - 29.3} \qquad (14)$$

At first this function appears more difficult to integrate than that for $R_\varepsilon(E)$, but two steps make this easier. Firstly, in Appendix 2 we prove that for these particular expressions of $R_\varepsilon(S)$ and $R_\varepsilon(E)$, the relation holds true that:

$$R_\varepsilon(E) - R_\varepsilon(S) = -1$$

Secondly, based on this, we prove in Appendix 3 that:

$$\frac{\int_{D_2}^{D_1} R_\varepsilon(S)dD}{D_1 - D_2} = \frac{I\big|_{D_2}^{D_1}}{D_1 - D_2} + 1$$

$$\text{i.e.} \qquad \frac{\int_{D_2}^{D_1} R_\varepsilon(S)dD}{D_1 - D_2} = \frac{\int_{D_2}^{D_1} R_\varepsilon(E)dD}{D_1 - D_2} + 1$$

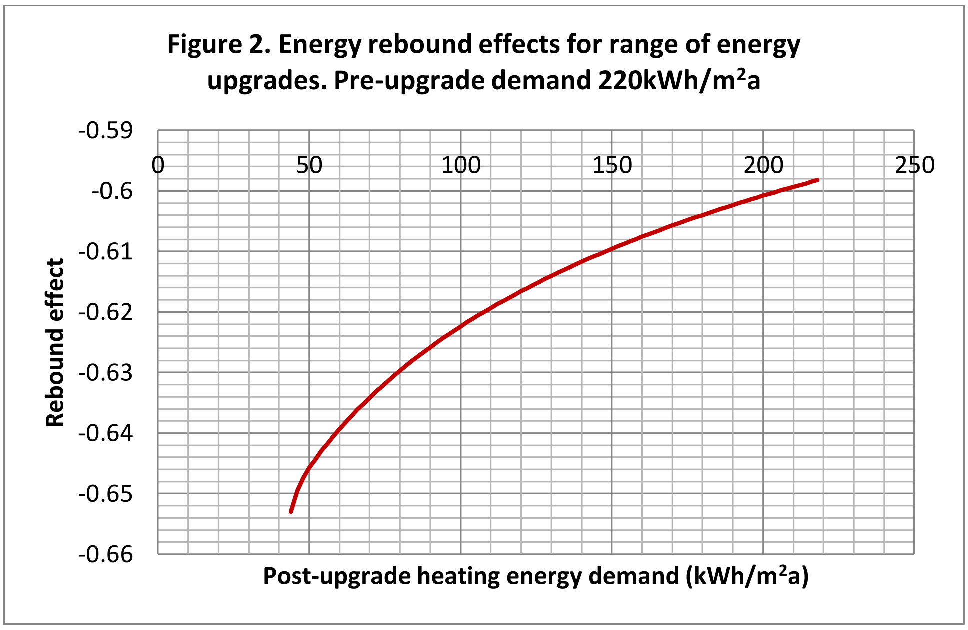
This proves coherence between the rebound effects for energy and for energy services averaged over any range of heating demand. To calculate the averaged energy services rebound effect we simply add 1 to the energy rebound effect.

4. Results

4.1 The case of an attempt to reduce consumption by 80%.

We first calculate the results for an upgrade that reduces the demand by 80%, from the average German demand of 220kWh/m²a. This is an interesting case because it relates to the stated aim of the German government to be achieved by 2050. The results for the energy rebound effect are displayed in Figure 2. Each point along the curve, starting at the right end, gives the energy rebound effect we would get for an upgrade from D =

220kWh/m^{2}a to the post-upgrade value of *D.* Moving to the left gives us energy rebound effects for successively higher upgrades, all starting from D = 220kWh/m^{2}a.

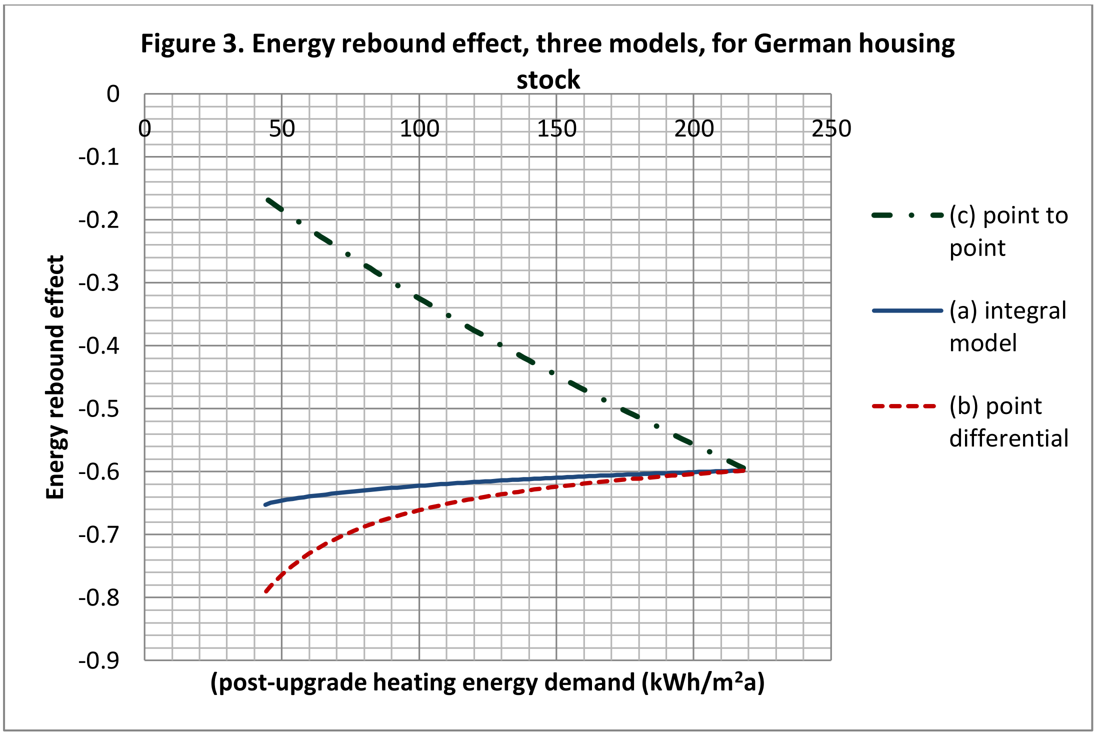


This shows, for example, that an upgrade that reduces demand from 220 to 200kWh/m^{2}a gives an energy rebound effect of -0.6008, meaning that 60% of the energy efficiency increase goes to reduce energy consumption while 40% goes to increase energy service take. For an upgrade from 220 to 100kWh/m^{2}a the figures are 62.24% and 37.76%. For an upgrade from 220 to 44kWh/m^{2}a (an 80% reduction in demand) the figures are 65.3% and 34.7%. (Note that this curve does *not* show rebound effects for upgrades starting at lower demand figures than 220). These figures are coherent, in that if we start from the energy services rebound effect we get the same results.

To illustrate the difference this integral-based approach makes to precision, Figure 3 gives the curve from Figure 2, labelled (a) in Figure 3, along with two others: (b) the single point rebound effect, i.e. the traditional formulation of the rebound effect for infinitesimal changes at all points along the curve, and (c) the point-to-point rebound effect, i.e. using $R_\varepsilon(E) = \frac{\Delta E}{E} \cdot \frac{\varepsilon}{\Delta \varepsilon}$ over the range 220 to 44kWh/m^{2}a.

Figure 3 illustrates that the non-integrated, single point rebound effect (b) shows significantly higher (numerically more negative) reductions in energy consumption as the demand diminishes. This is of little practical use, in itself, as nobody in the real world is likely to be interested in infinitesimal improvements in energy efficiency. It can lead to a mistake, however, namely thinking that an upgrade over the full span of (for example) 220 – 44kWh/m^{2}a will produce the rebound effect that actually only holds true for an infinitesimal energy efficiency increase at a demand of 44kWh/m^{2}a. In this case the error is 21%.

The biggest error, however, is in (c), the point-to-point rebound effect. It will be recalled from Section 2.3 that this is calculated simply by assuming that $\frac{\Delta E}{E} \cdot \frac{\varepsilon}{\Delta \varepsilon}$, for large changes in E and ε, gives the same result as $\frac{\partial E}{\partial \varepsilon} \cdot \frac{\varepsilon}{E}$. The error is considerable: -0.1654 compared with -0.6530. We note that the error becomes progressively larger the higher the magnitude of the energy efficiency upgrade. Further, as we saw in 2.2, starting such a calculation with the energy *services* rebound gives equally incoherent results.



Returning to the integrated rebound effect calculations, we also note that reducing the demand by 80% will not reduce consumption by 80%. Instead it reduces consumption from 147.7 to 50.0kWh/m²a, a reduction of 66%. For an 80% reduction in *actual* average consumption, the average post-upgrade consumption would need to be just under 30kWh/m²a. Inverting equation (1) to calculate *D* from this value of *E* shows that the average post-upgrade demand would have to be just under 25kWh/m²a to achieve a real 80% reduction in consumption. The computer programme as coded can integrate for values of *D* as low as 28kWh/m²a, but below that level over 170 terms are needed to achieve convergence of the infinite series, and at this point the factor (n-1!), i.e. 169!, is too large for the computer to handle. For an upgrade from a demand of 220 to 28kWh/m²a the energy rebound effect is -0.6667, and the trend is for it to be increasing sharply, so we would expect a value of around -0.7 for an upgrade from 220 to 25kWh/m²a.

An incidental issue, however, is the question of how plausible it is to maintain that a reduction to an average demand of 25kWh/m²a is possible for the German housing stock (see discussion in Galvin, 2010; Jakob, 2006).

4.2 Fuel-poor and high-consuming households

For the consumption/demand curve given in equation (1) the energy rebound effect for upgrades from a demand of 220kWh/m²a to any demand level down to 44 kWh are given in Figure 2 as noted above. As explained there, this curve is the best estimate we have for how *average* consumption varies with demand. But not all households consume the average heating energy for their dwelling's demand, and it is interesting to ask what magnitude of rebound effects could be expected from households with other consumption/demand curves. Greller et al. (2010) show that although there is a wide range of actual consumption *E* for each specific demand value *D,* the shape and spread of the distribution of *E* at each value of *D* is fairly consistent (see especially Figure 1., p. 2 in Greller et al., 2010). Hence we can consider a range of consumption/demand curves, with values of *E* in various proportions to each other.

To begin with, we consider 'fuel-poor' households (see Milne and Boardman, 2000, for a discussion of fuel poverty) which consume half the national average for any specified demand. The consumption/demand relation is therefore:

$$E = 6D^{0.499} - 14.65 \qquad (15)$$

Hence $P = 6$, $Q = 0.499$, $T = 14.65$, so that from equations (8, 9 and 10) we have

$$a = 4.893 \quad b = -0.499 \quad c = -2.004$$

This gives the identical energy rebound effect equation as that for the average household, i.e.

$$R_{\varepsilon}(E) = (4.983D^{-0.499} - 2.004)^{-1}$$

This non-intuitive result is due to the fact that, while these homes reduce less energy in absolute terms after an upgrade than 'average' homes do, the quantity of energy they consumed prior to the upgrade was also less, in the same proportion. Hence, in mathematical terms at least, no basis comes to light here for the concern expressed by such authors as Jenkins et al. (2011: 15) that we can expect larger rebound effects from fuel-poor households (or nations). There may be credible *social* grounds for such a concern, but the mathematics of the rebound effect, in this form, cannot be adduced to support it.

Similarly, high-consuming households that consume, say, twice as much heating fuel as the average for any specific demand will also show the same rebound effect as average and fuel-poor households. Of course, their absolute level of consumption will be twice as high after an upgrade as the average, just as the absolute level for a fuel-poor household will be half the average. This will be of interest to policy actors seeking to bring down consumption in upgraded homes (they might try to persuade high-consuming householders to consume less) and to actors seeking to eliminate fuel poverty (they might subsidise poorer households' fuel bills, for example). In this sense the rebound effect calculation methodology offered here could be a help to policymakers to anticipate possible post-upgrade social interventions.

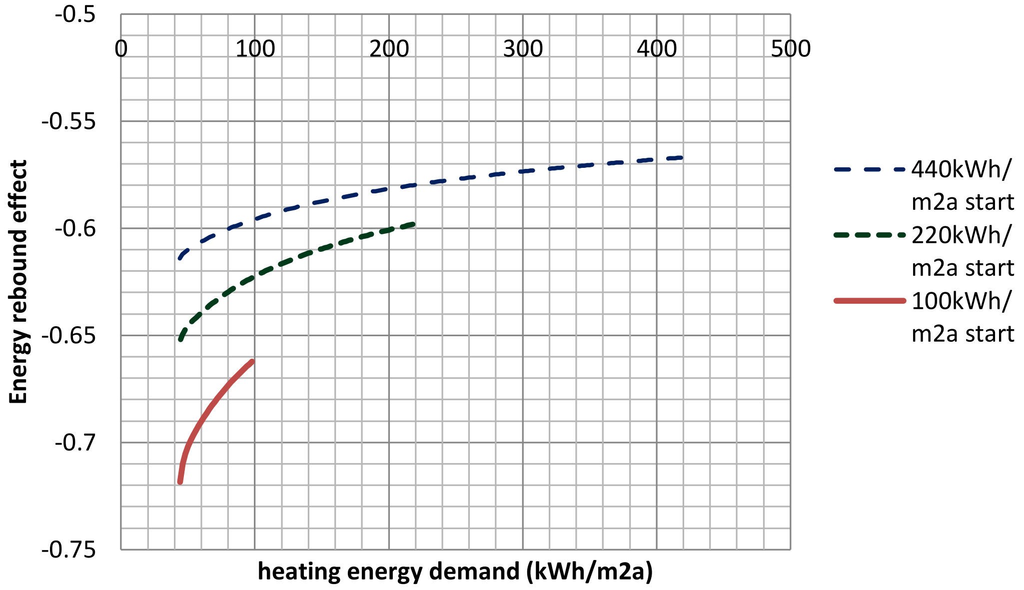
Nevertheless, it must also be emphasised that people do not usually behave consistently with pure mathematics, and might do unpredictable things after a large energy efficiency upgrade. But the mathematics can help social scientists identify which post-upgrade behaviours might best be explained by factors that lie outside of what the rebound effect parameters cover. For example, if a previously high-consuming household (as defined above) consumes significantly less than twice the new demand figure after an upgrade, the analysis here suggests that this change requires a social explanation.

4.3 Cases with different pre-upgrade heating energy demands

Although there is no difference in the rebound effect for high-consuming, average and fuel-poor households as defined above, it should be noted that there is a difference for the same cohort but with different pre-upgrade demand levels. In Section 4.1 we showed rebound effects for a range of depths of upgrade, all starting from a demand of 220kWh/m^{2}a. We now show energy rebound effects along the same consumption/demand curve, but starting with (a) 420kWh/m^{2}a, (b) 220kWh/m^{2}a, as above, and (c) 100 kWh/m^{2}a. In all cases these dwellings are upgraded to a demand of 44kWh/m^{2}a. These are displayed in Figure 4.

As seen in Figure 4, upgrading a dwelling with a higher pre-upgrade heating demand produces a smaller (less negative) energy rebound effect than for a dwelling with a lower pre-upgrade demand. In other words, its energy services rebound effect is significantly higher. A larger portion of the energy efficiency improvement goes to increasing energy services, than for dwellings on the same consumption/demand curve that have lower pre-upgrade demand.

Figure 4. Energy rebound effects for 'average' German dwellings of different pre-upgrade demand levels



So, for example, upgrading a dwelling with a pre-upgrade demand of 440kWh/m²a, only as far as a new demand of 220kWh/m²a, will bring it to the same position on the demand curve as a dwelling that was always at 220kWh/m²a (the two dwellings will now both have the same *E* and *D*), but the first dwelling will have undergone a rebound effect in order to get there. Likewise, while all three dwellings end up with a demand of 44kWh/m²a, their energy rebound effects are -0.61, -0.65 and -0.72 respectively. Expressed in terms of energy services take, their rebound effects are 39%, 35% and 28% respectively. This is the case even though they all end up consuming the same quantity of heating energy.

This is the sense in which higher rebound effects can be expected from upgrades of thermally inferior homes. This is not an issue of low-income households having larger rebounds, as it is often popularly claimed. Rather, higher rebound effects are to be expected from upgrades of thermally worse dwellings, regardless of who lives in them or how heavily they habitually consume.

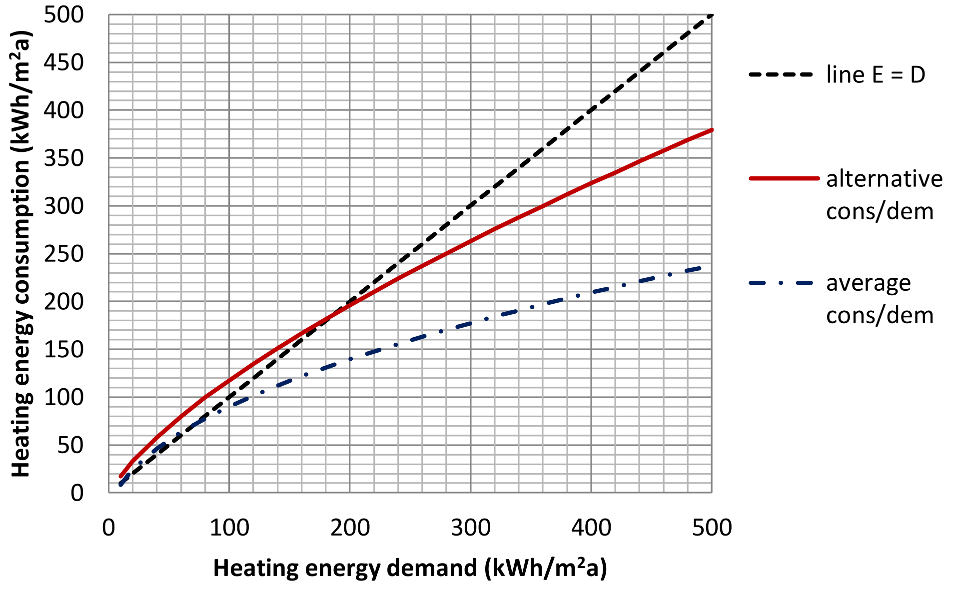
4.4 Cases with steeper consumption/demand curves

Finally, we examine the case of a set of households whose consumption/demand ratio more nearly approximates the line *E* = *D,* i.e. the index of *D* is closer to 1 than to 0.5. One such model is given by the relation:

$$E = 5D^{0.7} - 8 \qquad (16)$$

The consumption/demand curve for this is displayed alongside that for the 'average' model, together with the line *E* =*D*, in Figure 5.

Figure 5. Alternative consumption/demand curve alongside average, and line for E=D

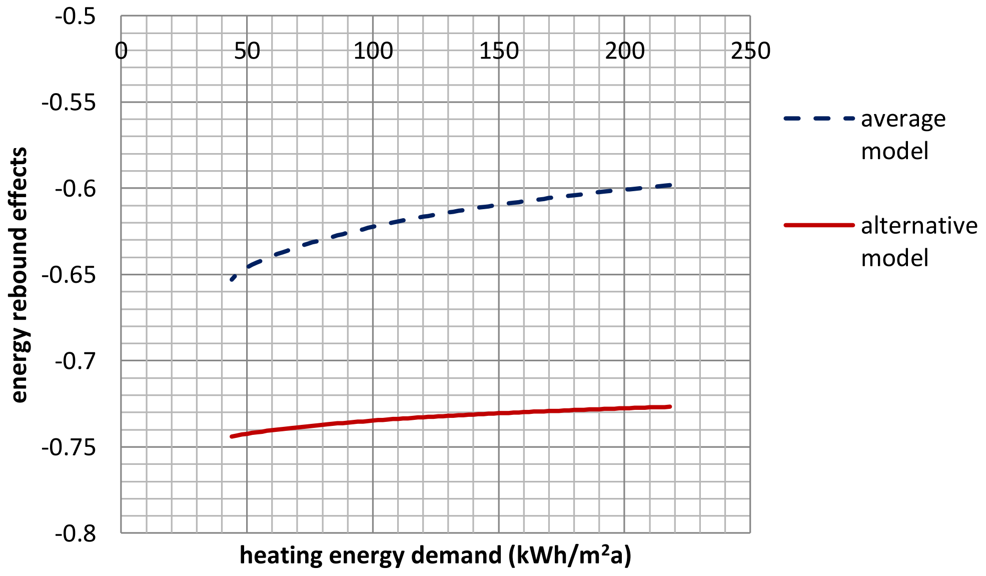


For this relation we get the values a = 2.286, b = -0.7, c = 1.429, giving the energy rebound curve:

$$R_\varepsilon(E) = \left(2.286D^{-0.7} - 1.429\right)^{-1} \qquad (17)$$

The energy rebound effect $R_\varepsilon(E)$ for an upgrade of such a dwelling from 220kWh/m²a to a range of demand levels down to 44kWh/m²a is displayed alongside that of the average model, in Figure 6.

Figure 6. Comparison of rebound effects for average model and alternative with consumption closer to demand



This shows that for the alternative case, a significantly greater proportion of the energy efficiency gain goes to reduction of energy consumption, than for the average case. The nearer the index of *D* is to 1.0, i.e. the nearer consumption/demand curve approximates the line E = D, the more the energy-efficiency gain is translated directly into gains in energy saving. However, in all the empirically derived models known to the author, the index of D is well under 0.6, and generally close to 0.5. Hence the alternative case here is of academic interest only. It illustrates that if more accurate empirical estimates of the consumption/demand curve are produced in the future, a the higher the index of D, the lower the energy services rebound effect and the higher the proportion of the energy efficiency increase is that goes to reducing energy.

5. Conclusions

This paper has shown what happens when the mathematics of the classical rebound effect are extended so that precise (theoretical) figures for the rebound effect can be obtained for energy efficiency upgrades of homes, using Germany's national housing stock as a case study.

The term 'rebound effect' usually denotes a shortfall in energy savings and/or an increase in energy service take following an energy efficiency upgrade, but the term is often used imprecisely in academic literature. However, formulations of the concept have clustered around a precise mathematical definition of the rebound effect, as the energy efficiency elasticity of energy services, with its correlate the energy efficiency elasticity of energy consumption. This conceptual precision has enabled some degree of stability and interchangeability between different sets of empirically derived results.

But this definition is structured as a partial differential, so that it only holds true for infinitesimal changes in energy consumption and energy service take associated with infinitesimal changes in energy efficiency. This makes its results internally inconsistent and incoherent for large changes, of the type we see in energy-efficiency upgrades of existing homes.

This paper has shown how this problem can be solved for these upgrades, using five steps. Firstly, a relation of the type E = f(D) is obtained, from empirical studies, for the average energy consumption E for each specific value of heating demand D. Secondly, this is transformed into a relation of the type E = f(ε), where ε is the heating efficiency of the dwelling. Thirdly, this relation is differentiated and the result multiplied by ε/E, to give the energy rebound effect relation $R_\varepsilon(E) = \frac{\partial E}{\partial \varepsilon} \cdot \frac{\varepsilon}{E}$. This is then transformed into a function in D, to give a precise figure for the energy rebound effect at any point along the consumption/demand curve. Finally, this function is integrated, and the definite integral between pre-upgrade and post-upgrade values of D is calculated and divided by the difference between the two values of D, to give the precise energy rebound effect for the entire upgrade.

The energy services rebound effect $R_\varepsilon(S)$ can be calculated from this result simply by adding 1.0 to it. The results are mathematically coherent and consistent with the properties of curvilinear functions. They tell us precisely what rebound effect would ensue if consumers behaved according to the consumption/demand model developed from empirical studies.

For the average German dwelling, with a demand of 220kWh/m^{2}a, reducing the demand by an average of 80% would lead to an energy rebound effect of -0.653, meaning that 65.3% of the energy efficiency improvement would go to reducing energy consumption, while 34.7% would go to increasing the take of energy services. An 80% reduction in energy *consumption* would not be achieved, as energy consumption would reduce, not from 220 to 44kWh/m^{2}a, but from 148 to 50kWh/m^{2}a, a reduction of 66.2%. To achieve an actual reduction in consumption of 80% this dwelling would have to be upgraded from its present demand of 220kWh/m^{2}a to 25kWh/m^{2}a. Here 70% of the energy efficiency gain would go to reducing energy consumption and the remaining 30% to increasing the take of energy services.

Rebound effects will be higher for dwellings with high pre-upgrade demand, than for those with lower pre-upgrade demand, regardless of the depth of the upgrade. For dwellings with the national average consumption at each specific demand value, i.e. situated on the consumption/demand curve $E = 12D^{0.499} - 20.3$, energy rebound effects will range from around -0.61 to -0.72 when these dwellings are upgraded to a demand of 44kWh/m^{2}a, depending on their pre-upgrade demand. This means the energy services rebound effect will range from 39% to 28%. The mathematics indicate that this is the range the German government needs to consider, in its aim to reduce consumption by 80%.

Fuel-poor homes, and high-consuming homes, produce the same rebound effect as homes with average consumption for each specific demand value. The idea that greater rebound effects can be expected from upgrading the homes of poor households is a myth, in terms of what the mathematics indicate. The cases that bring greater rebound effects are those with the highest pre-upgrade demand, as outlined above. These are usually large, old, detached or semi-detached homes with few occupants per m^2 of living area.

Different constellations of rebound effects would occur for households that follow consumption/demand curves with indexes of D that differ from 0.499. However, the author has not yet found a dataset that gives an index which deviates far from this value.

These findings could be useful for policymakers attempting to estimate the likely energy savings from programmes to upgrade national housing stock, for example to reduce heating energy consumption by 80%.

Nevertheless, it must be emphasised that these results are reliable only mathematically, given the model of consumption/demand derived empirically. For example, if a country were to succeed in reducing the heating energy consumption of its housing stock by 80%, this would be a massive socio-technical transformation, and might result in cultural shifts that lead to unforeseen changes in consumption patterns. The actual effects of upgrades can only be known empirically, from household-by-household investigation. However, it is suggested here that such investigation would benefit from knowing what the mathematically calculated effects of an upgrade are, so as to be better able to identify rogue or non-neutral shifts in consumption habits. In any case, if we have a potentially precise mathematical tool for processing empirical data, nothing will be lost by taking that tool to a higher level of precision, even if the data is itself subject to imprecision and uncertainty.

Appendix 1. Integrating the energy rebound effect for the consumption/demand curve of German home heating

Developing an expression for the integral of the Energy Rebound Effect $R_\varepsilon(E)$

Let $\quad I = \int (ax^b + c)^{-1} dx \quad where\ x = $ calculated heating demand

Using: $\quad \int u dv = uv - \int v du$

Let $\quad u = (ax^b + c)^{-1} \rightarrow du = -abx^{-1}(ax^b + c)^{-2} dx$

Let $\quad dv = 1 \cdot dx \quad \rightarrow v = x$

$$\rightarrow \quad I = x(ax^b + c)^{-1} + \int x \cdot abx^{b-1}(ax^b + c)^{-2} dx$$

$$= x(ax^b + c)^{-1} + \int abx^b(ax^b + c)^{-2} dx \qquad (1)$$

Let $J = \int abx^b(ax^b + c)^{-2}dx$

Let $u_1 = (ax^b + c)^{-2} \rightarrow du_1 = -2abx^{b-1}(ax^b + c)^{-3}dx$

Let $dv_1 = abx^b dx \rightarrow v_1 = \dfrac{abx^{b+1}}{b+1}$

$$\rightarrow J = \frac{abx^{b-1}}{b+1} \cdot (ax^b + c)^{-2} + \int \frac{abx^{b+1}}{b+1} \cdot 2abx^{b-1}(ax^b + c)^{-3}dx$$

$$= \frac{abx^{b-1}}{b+1} \cdot (ax^b + c)^{-2} + \int \frac{2a^2b^2x^{2b}}{b+1} \cdot 2abx^{b-1}(ax^b + c)^{-3}dx \quad (2)$$

Let $K = \int \dfrac{2a^2b^2x^{2b}}{b+1} \cdot 2abx^{b-1}(ax^b + c)^{-3}dx$

Let $u_2 = (ax^b + c)^{-3} \rightarrow du_2 = -3abx^{b-1}(ax^b + c)^{-4}dx$

Let $dv_2 = \dfrac{2a^2b^2x^{2b}}{b+1}dx \rightarrow v_2 = \dfrac{2a^2b^2x^{2b+1}}{(b+1)(2b+1)}$

$$\rightarrow K = \frac{2a^2b^2x^{2b+1}}{(b+1)(2b+1)} \cdot (ax^b + c)^{-3} + \int \frac{2a^2b^2x^{2b+1}}{(b+1)(2b+1)} \cdot 3abx^{b-1}(ax^b + c)^{-4}dx$$

$$= \frac{2a^2b^2x^{2b+1}}{(b+1)(2b+1)} \cdot (ax^b + c)^{-3} + \int \frac{6a^3b^3x^{3b}}{(b+1)(2b+1)} \cdot (ax^b + c)^{-4}dx \quad (3)$$

Let $L = \int \dfrac{6a^3b^3x^{3b}}{(b+1)(2b+1)} \cdot (ax^b + c)^{-4}dx$

Let $u_3 = (ax^b + c)^{-4} \rightarrow du_3 = -4abx^{b-1}(ax^b + c)^{-5}dx$

Let $dv_3 = \dfrac{6a^3b^3x^{3b}}{(b+1)(2b+1)}dx \rightarrow v_3 = \dfrac{6a^3b^3x^{3b+1}}{(b+1)(2b+1)(3b+1)}$

$$\rightarrow L = \frac{6a^3b^3x^{3b+1}}{(b+1)(2b+1)(3b+1)} \cdot (ax^b + c)^{-4} + \int \frac{6a^3b^3x^{3b+1}}{(b+1)(2b+1)(3b+1)} 4abx^{b-1}(ax^b + c)^{-5}dx$$

$$= \frac{6a^3b^3x^{3b+1}}{(b+1)(2b+1)(3b+1)} \cdot (ax^b + c)^{-4} + \int \frac{24a^4b^4x^{4b}}{(b+1)(2b+1)(3b+1)} (ax^b + c)^{-5}dx \quad (4)$$

Now combining (1), (2), (3) and (4)

$$I = x(ax^b + c)^{-1} + \frac{abx^{b-1}}{b+1} \cdot \left(ax^b + c\right)^{-2} + \frac{2a^2b^2x^{2b+1}}{(b+1)(2b+1)} \cdot \left(ax^b + c\right)^{-3}$$

$$+ \frac{6a^3b^3x^{3b+1}}{(b+1)(2b+1)(3b+1)} \cdot \left(ax^b + c\right)^{-4} \quad \ldots\ldots etc. \quad (5)$$

This can alternatively be expressed as:

$$I = \sum_{n=1}^{\infty} \frac{(n-1)!\,(ab)^{n-1}x^{(n-1)b+1}(ax^b + c)^{-n}}{[nb+1][(n-1)b+1][(n-2)b+1]\ldots[0 \times b + 1]} \quad (6)$$

Appendix 2. Proving the identity for these particular demand curves

The energy services rebound effect relation is:

$$R_\varepsilon(S) = \frac{6.012D^{0.499} - 29.3}{12D^{0.499} - 29.3}$$

$$= \frac{6.012D^{0.499}}{12D^{0.499} - 29.3} - \frac{29.3}{12D^{0.499} - 29.3}$$

$$= \frac{6.012D^{0.499} - 5.988D^{0.499} + 5.988D^{0.499}}{12D^{0.499} - 29.3} - \frac{29.3}{12D^{0.499} - 29.3}$$

$$= \frac{12D^{0.499}}{12D^{0.499} - 29.3} - \frac{5.988D^{0.499}}{12D^{0.499} - 29.3} - \frac{29.3}{12D^{0.499} - 29.3}$$

$$= \frac{12D^{0.499} - 29.3}{12D^{0.499} - 29.3} - \frac{5.988D^{0.499}}{12D^{0.499} - 29.3}$$

$$= 1 + R_\varepsilon(E)$$

Hence $R_\varepsilon(E) - R_\varepsilon(S) = -1$

In other words, the relationship between the energy services and the energy rebound effects inherent in their definition holds true for these formulations of $R_\varepsilon(S)$ and $R_\varepsilon(E)$.

Appendix 3. Integrating the energy services rebound effect

Let $M = \dfrac{\int_{D_2}^{D_1} R_\varepsilon(S)dD}{D_1 - D_2}$

Now $R_\varepsilon(E) - R_\varepsilon(S) = -1$ (see Appendix 2)

$\rightarrow R_\varepsilon(S) = R_\varepsilon(E) + 1$

$$\rightarrow M = \frac{\int_{D_2}^{D_1} [R_\varepsilon(E) + 1]dD}{D_1 - D_2}$$

$$= \frac{[I + D]|_{D_2}^{D_1}}{D_1 - D_2} \quad \text{where } I = \int (aD^b + c)^{-1} dD \quad \text{(see Appendix 1)}$$

$$= \frac{I|_{D_2}^{D_1}}{D_1 - D_2} + \frac{D_1 - D_2}{D_1 - D_2}$$

$$= \frac{I|_{D_2}^{D_1}}{D_1 - D_2} + 1$$

Hence to obtain the average energy services rebound effect over any range $D_1 - D_2$ on the consumption/demand curve we simply add 1 to the average energy rebound effect for that range.

References

Berkhout P, Muskens J, Velthuijsen J (2000) Defining the rebound effect. Energy Policy 28 (6–7): 425–432

Bonino D, Corno F, De Russis L (2012) Home energy consumption feedback: A user survey. Energy and Buildings 47:383–393

Demanuele C, Tweddell T, Davies M (2010) Bridging the gap between predicted and actual energy performance in schools. World Renewable Energy Congress XI 25-30 September 2010, Abu Dhabi, UAE.

DENA (2012) Der dena-Gebäudereport 2012: Statistiken und Analysen zur Energieeffizienz im Gebäudebestand. Berlin: Deutsche Energie-Agentur. Available online via: http://www.dena.de/publikationen/gebaeude/report-der-dena-gebaeudereport-2012.html accessed 20 February, 2012

DIN (2003) DIN 4108-2:2003-07 Wärmeschutz und Energie-Einsparung in Gebäuden: Mindestanforderungen an den Wärmeschutz. Beuth-Verlag, Berlin

Deurinck M, Saelens D, Roels S (2012) Assessment of the physical part of the temperature takeback for residential retrofits. Energy and Buildings 52: 112–121

Druckman A, Chitnis M, Sorrell S, Jackson T (2011) Missing carbon reductions? Exploring rebound and backfire effects in UK households. Energy Policy 39: 3572–3581

Erhorn H (2007) Bedarf – Verbrauch: Ein Reizthema ohne Ende oder die Chance für sachliche Energieberatung? Fraunhofer-Institut für Bauphysik, Stuttgart (available at: http://www.buildup.eu/publications/1810) accessed 20 November 2011

Galvin R (2010) Thermal upgrades of existing homes in Germany: The building code, subsidies, and economic efficiency. Energy and Buildings 42: 834–844

Giraudet L, Guivarch C, Quirion P(2012) Exploring the potential for energy conservation in French households through hybrid modelling. Energy Economics 34: 426–445

Greller M, Schröder F, Hundt V, Mundry B, Papert O (2010) Universelle Energiekennzahlen für Deutschland – Teil 2: Verbrauchskennzahlentwicklung nach Baualtersklassen. Bauphysik, 32(1), 1–6

Guerra Santin O, Itard L, Visscher H (2009) The effect of occupancy and building characteristics on energy use for space and water heating in Dutch residential stock. Energy and Buildings 41: 1223–1232

Haas R, Biermayr R (2000) The rebound effect for space heating: Empirical evidence from Austria. Energy Policy 28: 403-410

Hinnells M (2008) Technologies to achieve demand reduction and microgeneration in buildings. Energy Policy 36: 4427–4433

Howden-Chapman P, Viggers H, Chapman R, O'Dea D, Free S, O'Sullivan K (2009) Warm homes: Drivers of the demand for heating in the residential sector in New Zealand. Energy Policy 37: 3387–3399

Jagnow K, Wolf D (2008) Technische Optimierung und Energieinsparung, OPTIMUS-Hamburg City-State.

Jakob M (2006) Marginal costs and co-benefits of energy efficiency investments: the case of the Swiss residential sector, Energy Policy 34: 172–187.

Jakob M, Haller M, Marschinski R (2012) Will history repeat itself? Economic convergence and convergence in energy use patterns. Energy Economics 34: 95–104

Jenkins J, Nordhaus T, Shellenberer S (2011) ENERGY EMERGENCE: REBOUND & BACKFIRE AS EMERGENT PHENOMENA. Breakthrough Institute.

Jin S (2007) The effectiveness of energy efficiency improvement in a developing country: Rebound effect of residential electricity use in South Korea. Energy Policy 35: 5622–5629

Kaßner R, Wilkens M, Wenzel W, Ortjohan J (2010) Online-Monitoring zur Sicherstellung energetischer Zielwerte in der Baupraxis, in Paper presented at 3. Effizienz Tagung Bauen+Modernisieren, Hannover, Germany, November, 19–20, 2010 (available at: http://www.energycheck.de/wp-content/uploads/2010/11/EFT_2010_ortjohann_2010-10-18.pdf) accessed on 18 November 2011

Knissel, J. and Loga, T. (2006) Vereinfachte Ermittlung von Primärenergiekennwerten. Bauphysik, 28(4): 270–277.

Loga T, Diefenbach N, Born R (2011) Deutsche Gebäudetypologie. Beispielhafte Maßnahmen zur Verbesserung der Energieeffizienz von typischen Wohngebäuden. Institute Wohnen und Umwelt, Darmstadt.

Madlener M, Hauertmann M (2011) Rebound Effects in German Residential Heating: Do Ownership and Income Matter? FCN Working Paper No. 2/2011, Energy Research Centre, RWTH-Aachen University

Milne G , Boardman B (2000) Making cold homes warmer: the effect of energy effciency improvements in low-income homes. Energy Policy 28: 411-424

Schipper L (2000) On the rebound: the interaction of energy effciency, energy use and economic activity. An introduction (editorial) Energy Policy 28: 351-353

Sorrel S (2007) The Rebound Effect: an assessment of the evidence for economy-wide energy savings from improved energy efficiency. A report produced by the Sussex Energy Group for the Technology and Policy Assessment function of the UK Energy Research Centre: UK Energy Research Centre.

Sorrel S, Dimitropoulos J (2008) The rebound effect: Microeconomic definitions, limitations and extensions. Ecological Economics 65: 636-649

Sorrell S, Dimitropoulos J, Sommerville M (2009) Empirical estimates of the direct rebound effect: A review. Energy Policy 37: 1356–1371

Sunikka-Blank M, Galvin R (2012) Introducing the prebound effect: the gap between performance and actual energy consumption. Building Research and Information 40: 260-273

Tighelaar C, Menkveld M (2011) Obligations in the existing housing stock: who pays the bill? Proceedings of the ECEEE 2011 Summer study on Energy Efficiency First: The Foundation of a Low-Carbon Society, pp. 353–363

Tronchin L, Fabbri K (2007) Energy performance building evaluation in Mediterranean countries: Comparison between software simulations and operating rating simulation. Energy and Buildings 40(7): 1176–1187

Yun B, Zhang J, Fujiwara A (2013) Evaluating the direct and indirect rebound effects in household energy consumption behavior: A case study of Beijing. Energy Policy (in press) http://dx.doi.org/10.1016/j.enpol.2013.02.024i